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JOURNAL OF THE AERONAUTICAL SCIENCES

CONTENTS

Contributors to This Issue iv

Flaps and the Take Off . . . . . G. V. LACHMANN 87

A Method for Determining the Ultimate Strength of a Column Swaged at the Ends . . . . . J. M. GWINN, JR. AND ROY A. MILLER 94

The Computation of the Apparent Mass of Dirigibles . . MAX M. MUNK 97

Dynamic Longitudinal Stability Charts for Design Use . . . L. E. ROOT 101

Optimum Flight Path in Air Transport Operation . W. C. ROCKEFELLER 109

The Scale of the Boundary Layer in the Atmosphere . . . . . ROBERT H. GODDARD AND ARNOLD M. KUETHE 115

Book Review 117

Our Wings Grow Faster . . . . . *Grover Loening*

Periodically Interrupted Flow Through Air Passages . ALBERT C. ERICKSON 118

Institute Notes 123

Letters to the Editor 125

Advertising vii

VOLUME 2 MAY, 1935 NUMBER 3

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JOURNAL OF THE AERONAUTICAL SCIENCES

Volume 2 MAY, 1935 Number 3

Flaps and the Take Off

G. V. LACHMANN, *Handley Page, Cricklewood, England*

(Received October 11, 1934)

AERODYNAMIC CHARACTERISTICS OF EXISTING FLAP TYPES

THE modern fast monoplane with its parasitic drag reduced to a minimum and with a sufficiently low span loading to ensure a good rate of climb and economical cruising is inconceivable without the aid of a device for steepening the gliding angle and for reducing the landing speed. Thus, with the development of fast and clean monoplanes, flaps have come into general use. The type which is mainly used in America is the split flap. Although the principle of the split flap has been known for many years it was only comparatively recently that the aerodynamic principle has been fully understood and appreciated. We know now that the lift increase obtained by the split flap is always associated with a considerable increase of drag.

The ideal flap should give a considerable lift increase without a correspondingly large increase of drag, if the take off is to be improved, and a large drag increase should be available when it is desired to steepen the gliding angle for landing. Compared with the normal airfoil, the split flap gives only a slight improvement in take off on account of the high drag associated with the increase of lift.

In Fig. 1 a comparison between the three representative flap types, slotted, split and ordinary flaps is given. Other types like the Junkers flap or the Zap flap can be considered as modifications of one of the standard types. The test VL was approximately 90 in each case.

THE HANDLEY PAGE SLOTTED FLAP

A wing section fitted with a slotted flap of the Handley Page type is shown in Fig. 2. This type of slotted flap is the result of considerable research work carried out in the Handley Page wind tunnel with a view to developing a flap giving practically no increase of profile drag in the neutral position, a considerable increase of lift with a small increase of drag with the flap pulled down about 20° to 30° and a large increase of lift and drag with the flap pulled down to approximately 50°.

Fig. 3 gives a polar diagram for a slotted flap only, with the flap set at various angles of incidence. It follows from this diagram that even with relatively small flap angles a fairly good lift increase over the normal section can be obtained without a too pronounced increase of profile drag.

There is a certain aerodynamic relationship between the slotted and the split flap. In both cases the effect is dependent on the reduction of pressure at the trailing edge of the main wing, as shown in Fig. 4. As burbling depends on the pressure gradient from leading edge to trailing

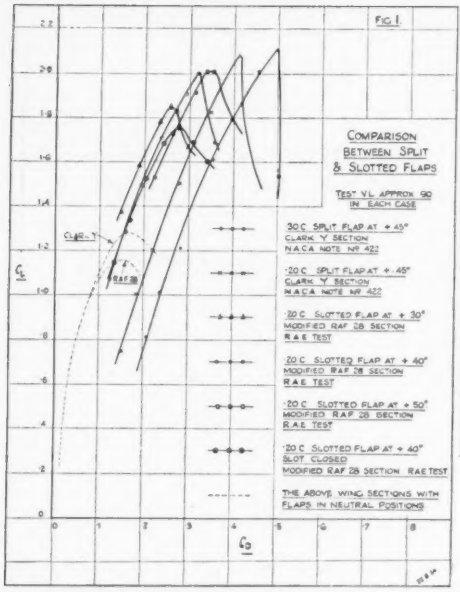
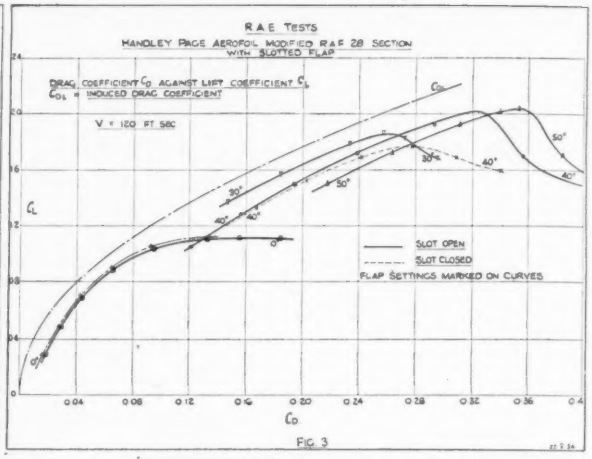
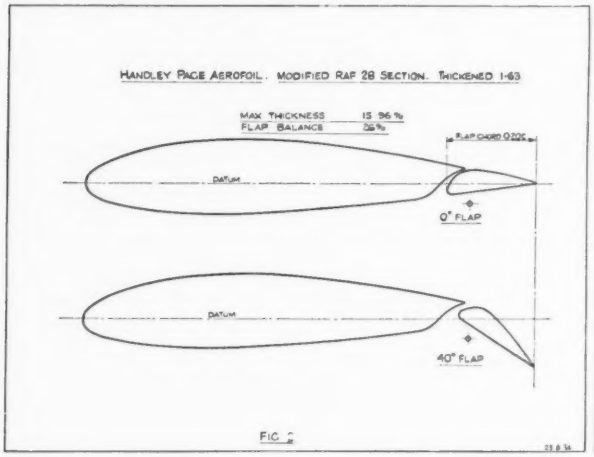
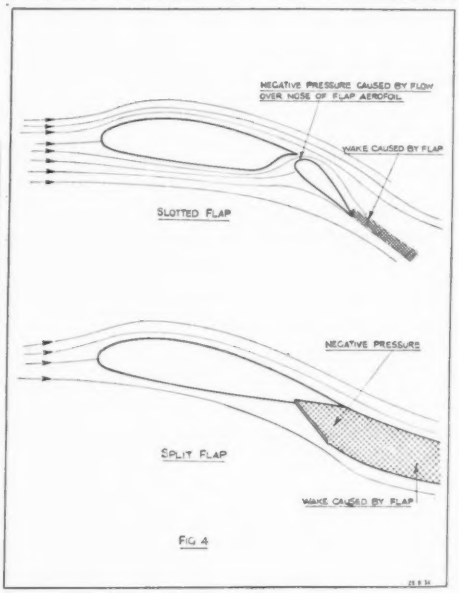


FIG 1
COMPARISON BETWEEN SPLIT & SLOTTED FLAPS
TEST VL APPROX 90 IN EACH CASE
·30 C SPLIT FLAP AT + 45° CLARK Y SECTION NACA NOTE Nº 422
·20 C SPLIT FLAP AT + 45° CLARK Y SECTION NACA NOTE Nº 422
·30 C SLOTTED FLAP AT + 30° MODIFIED RAF 28 SECTION R.A.E. TEST
·20 C SLOTTED FLAP AT + 40° MODIFIED RAF 28 SECTION R.A.E. TEST
·20 C SLOTTED FLAP AT + 50° MODIFIED RAF 28 SECTION R.A.E. TEST
·20 C SLOTTED FLAP AT + 40° SLOT CLOSED MODIFIED RAF 28 SECTION R.A.E. TEST
THE ABOVE WING SECTIONS WITH FLAPS IN NEUTRAL POSITIONS
CLARK Y
RAF 28
CL
CD


R.A.E. TESTS
HANDLEY PAGE AEROFOIL MODIFIED RAF 28 SECTION WITH SLOTTED FLAP
DRAG COEFFICIENT CD AGAINST LIFT COEFFICIENT CL
CDi = INDUCED DRAG COEFFICIENT
V = 120 FT SEC
SLOT OPEN
SLOT CLOSED
FLAP SETTINGS MARKED ON CURVES
FIG 3





Figs. 2 and 3, Royal Air Force Official, Crown Copyright Reserved.

edge, it is obvious that by lowering the pressure at the trailing edge, the pressure at the leading edge can also be considerably lowered, with the result that the front wing experiences a much higher lift than it would if the pressure were equalised at the trailing edge.

In the case of the split flap, the low pressure region is caused "statically" by the formation of a dead air region, which, of course, results in a considerable increase in profile drag, while the low pressure region is formed in the case of the slotted flap "dynamically" by the acceleration of flow passing over the nose of the airfoil which forms the rear flap.

This effect is generally referred to as "slot effect." In the case of the ordinary slotted wing where a small auxiliary airfoil is arranged in front of a main wing, it is solely the front wing

which experiences the lift increase due to the fact that it is situated in the accelerated and converging flow around the nose of the main wing. The same effect for a wing fitted with slotted flaps is clearly indicated in Fig. 5 where pressure plottings are shown for the main wing and the rear wing. The attached table also gives the values of C_N for front wing and rear flap. The extraordinary high value of C_N for the front wing is most remarkable while the coefficients of the normal force on the rear flap remain within the values which one would normally expect for an airfoil of this shape. Fig. 6 shows the effect of closing the slot. The pressure now being positive at the trailing edge of the main wing, it is impossible for the low pressure at the nose of the main wing to reach the same low values as before.

The C.P. of the rear flap remains fairly steady at 38 percent of the flap chord and it is therefore possible to balance the flap to a very high degree which, of course, facilitates operation and makes the use of mechanical devices such as hydraulic jacks or electric motors unnecessary for aircraft of medium size.

The Three Phases of the Take Off

The effect of the flaps on landing is generally known and need not be further discussed. It will be shown, however, that the take off of a specific modern airplane can be materially improved by the use of slotted flaps. In order to show the effect of slotted flaps and split flaps on the take off, a method is used wherein the take off process is analyzed in three phases.

(a) Acceleration up to unstick speed.
(b) Change of flight path which occurs at the end of the unstick run.
(c) Climb over the obstacle.

(a) Unstick Run

The late Dr. M. Schrenk has shown in a valuable paper[1] that the integration for the unstick run can be simply carried out by using the energy equation

$$Pdl = \frac{W}{g} d\left(\frac{v}{2}\right)^2$$

now

$$\frac{v^2}{2} = \frac{q}{\rho},$$

hence,

$$dl = \frac{W}{\gamma} \frac{dq}{P},$$

where P = accelerating force = $T - D - F$,

W = weight,
T = propulsive thrust,
D = drag,
F = friction,
l = length of run,
$q = \frac{1}{2}\rho v^2$ = dynamic pressure.

Introducing Alayrac's[2] approximation of a linear function between T and q, a linear law is obtained for the accelerating force P.

$$P = P_0 - \frac{P_0 - P_1}{q_1} \cdot q,$$

hence

$$dl = \frac{W}{\gamma} \cdot \frac{d_q}{P_0 - \frac{P_0 - P_1}{q_1} \cdot q},$$

$$l_1 = \frac{W}{\gamma} \cdot \frac{q_1}{P_0 - P_1} \log \frac{P_0}{P_1},$$

where P_0 = accelerating force at the beginning of the unstick run,

$$P_0 = T_0 - \mu W \quad \text{(See Fig. 7)},$$

where "μ" is the coefficient of rolling friction, and where P_1 = accelerating force at the instant of unstick.

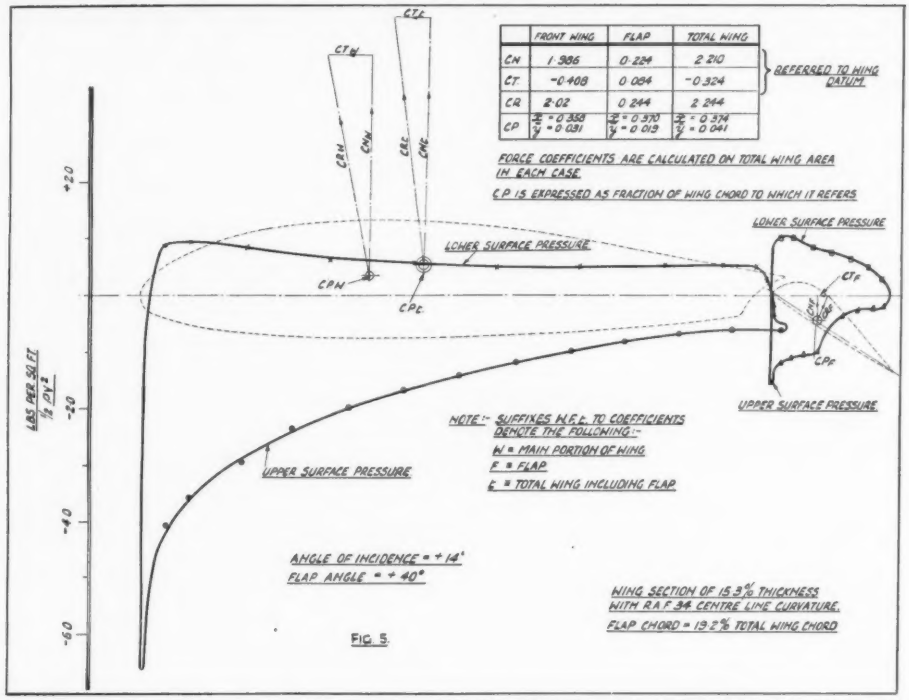
$$P_1 = T_1 - D_1.$$

(b) Change of flight path

At the end of the unstick run the pilot increases the angle of incidence and in order to attain an upward inclination of the flight path at an angle θ, the aircraft has to fly through an arc $R\theta$. The projection of this arc on the horizontal means another loss of distance and the

[1] Martin Schrenk, *Abflug und Schraubenschub*, 305 Bericht der D.V.L., ZFM, 1932, Nr. 21. For translation see N.A.C.A. T.M. 703.

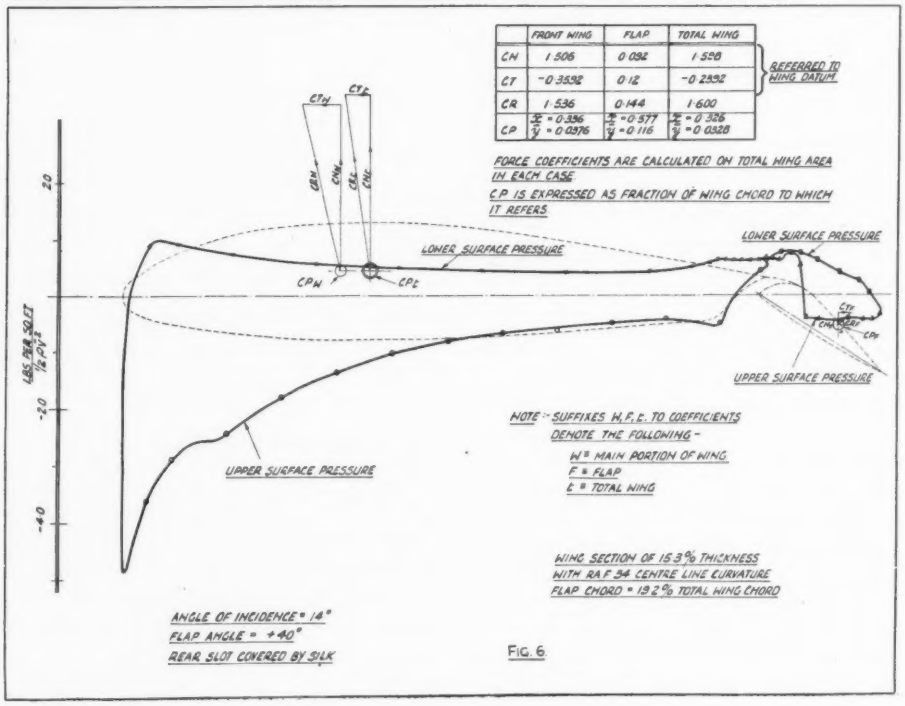
[2] Alayrac, *Etude sur le vol ascendant*, La Technique Aeronautique, 10, p. 41.



	FRONT WING	FLAP	TOTAL WING	
CN	1·986	0·224	2·210	REFERRED TO WING DATUM
CT	-0·408	0·084	-0·324	
CR	2·02	0·244	2·244	
CP	$\frac{x}{c}$ = 0·358 $\frac{y}{c}$ = 0·031	$\frac{x}{c}$ = 0·370 $\frac{y}{c}$ = 0·013	$\frac{x}{c}$ = 0·374 $\frac{y}{c}$ = 0·041	

FORCE COEFFICIENTS ARE CALCULATED ON TOTAL WING AREA IN EACH CASE.
C.P. IS EXPRESSED AS FRACTION OF WING CHORD TO WHICH IT REFERS.

FIG. 5.



	FRONT WING	FLAP	TOTAL WING	
CN	1·506	0·032	1·528	REFERRED TO WING DATUM
CT	-0·3592	0·12	-0·2392	
CR	1·536	0·144	1·600	
CP	$\frac{x}{c}$ = 0·336 $\frac{y}{c}$ = 0·0976	$\frac{x}{c}$ = 0·577 $\frac{y}{c}$ = 0·116	$\frac{x}{c}$ = 0·326 $\frac{y}{c}$ = 0·0928	

FORCE COEFFICIENTS ARE CALCULATED ON TOTAL WING AREA IN EACH CASE.
C.P. IS EXPRESSED AS FRACTION OF WING CHORD TO WHICH IT REFERS.

FIG. 6.

smaller the radius R of the arc, the smaller will be the loss. In order to describe this arc, a centripetal force is necessary:

$$\frac{W}{g}\frac{v^2}{R}=C_L'S\frac{\rho v^2}{2}-W=S\frac{\rho v^2}{2}(C_L'-C_L),$$

where C_L' = lift coefficient attained during the change of flight path,
C_L = lift coefficient at the instant of unstick.

The radius R therefore becomes:

$$R=\frac{2}{\gamma}\frac{W}{S}\left(\frac{1}{C_L'-C_L}\right) \quad \text{(See Fig. 8).}$$

It is the most reasonable assumption to put:

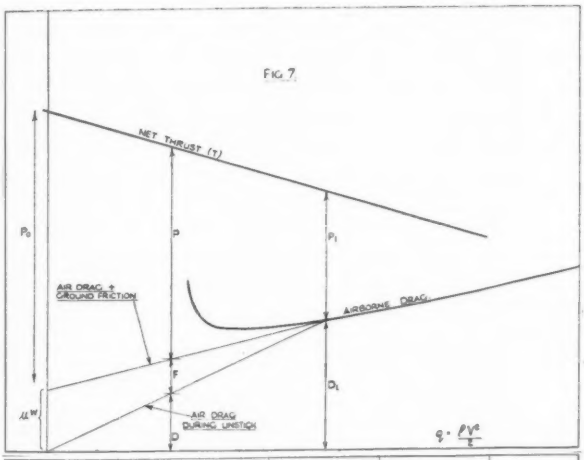
$C_L'=C_L$ max.,
$C_L=C_{L_1}$ = Lift coefficient corresponding to the attitude of the aircraft at the instant of unstick.

The total loss of distance $2l_2$ then becomes.

$$2l_2=\theta\frac{2}{\gamma}\frac{W}{S}\left(\frac{1}{C_L\,\text{max.}-C_{L_1}}\right)$$

and since $\sin\theta=\theta=(T_1-D_1)/W$ one finally obtains:

$$l_2=\frac{T_1-D_1}{W\cdot\gamma}\,\frac{q_1}{C_L\,\text{max.}/C_{L_1}-1}.$$



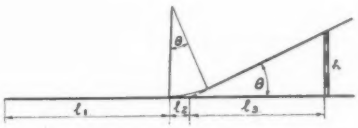
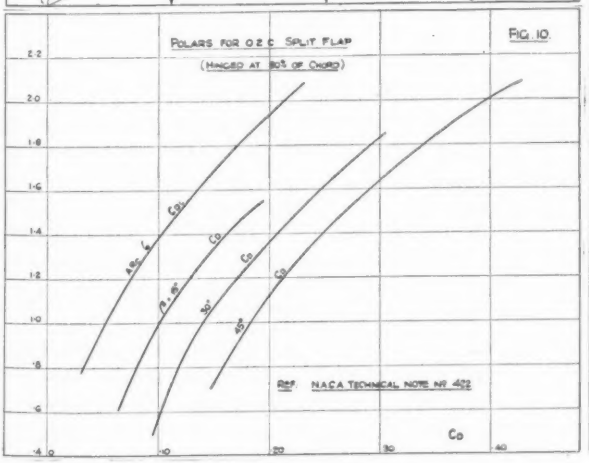
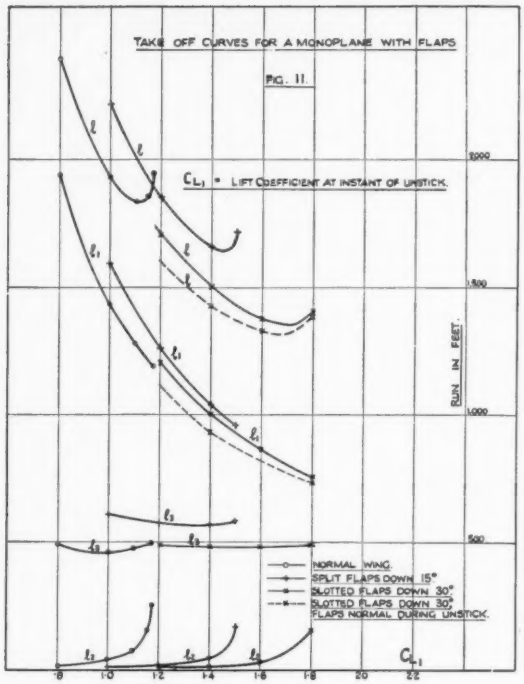


FIG. 8


POLARS FOR 0.2 C SPLIT FLAP
(HINGED AT 80% OF CHORD)
FIG. 10
REF. NACA TECHNICAL NOTE NO. 402
Cp
Cp
Cp



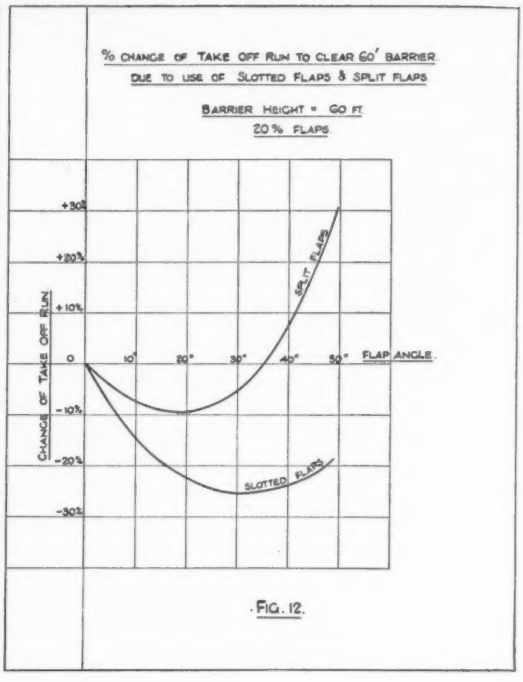


FIG. 12.

(c) Climb

The third phase is pure climb and the best climbing angle depends mainly on power loading and span loading.

$$l_3 = \frac{h}{\tan\theta} = \frac{Wh}{T_1 - D_1}.$$

Influence of Slotted and Split Flaps on the Take Off of a Specific Airplane

The following example for an aircraft of modern design characteristics has been worked out.

Total Weight	11,400 lbs.
Span	65 feet.
w	19 lbs./sq. ft.
Parasitic Drag Area	8.5 sq. ft.
Flap Chord/Wing Chord	0.2
Barrier Height	60 feet.
Take off Thrust $T_0 = 2950 - 23.5\ q$ lb.	

The aerodynamical characteristics of the wing with split flaps were obtained from N. A. C. A. Technical Note 422 while the characteristics for the wing with slotted flaps were taken from R. A. E. Tests on a Handley Page airfoil. Fig. 9 shows the actual C_L max. and C_{D_0} curves (as used in the calculation) plotted against flap angle β. With the slotted flap, the profile drag for a given flap setting remains sensibly constant whereas with the split flap, there is a considerable increase of profile drag with incidence. The polar curves for the split flap are shown in Fig. 10.

The three phases of the take off were calculated on the basis of the formulae given above and the lengths giving the minimum run to clear the barrier have been plotted in Fig. 11 as a function of C_{L_1}, that is, the lift coefficient for unstick speed. l_1 gives the length of the unstick run, l_2 the length required for attaining the flight path angle θ, and l_3 is the length required to climb to the height of the barrier. The total run is represented by the sum of $l_1+l_2+l_3$. In comparing the three components of the total run it is obvious where the flapped aircraft, especially the machine fitted with slotted flaps, has the advantage over the normal machine.

Owing to its higher lift coefficient and the

resulting lower unstick speed, the flapped aircraft has a much shorter unstick run. The distance l_2 representing the length of the arc through which the aircraft has to be turned at the end of the unstick run is, in the case of the flapped aircraft, only a fraction of the amount required for the unflapped machine.

The normal aircraft without flaps has a slight advantage only in regard to the distance l_3 required for climbing. Fig. 12 shows the change in total take off run caused by the flaps when set at various angles. Taking the optimum values, the split flap decreases the total run by 9.5 percent, whereas the slotted flap decreases the run by 25.5 percent.

A slightly shorter take off can be obtained if the flaps are neutral at the beginning of the run and are pulled down at the end of the unstick run. The reduction in unstick run and total run are shown by dotted lines. See Fig. 11.

Apart from the reduction in take off run, Fig. 11 reveals another important point. It shows that the minima of the curves representing the total runs are very pronounced for the unflapped machine and the machine with split flaps whereas the curve is very flat for the machine fitted with slotted flaps. In other words, taking off a machine with slotted flaps at minimum distance requires less piloting skill.

It is to be concluded, therefore, that slotted flaps materially improve the take off of the particular airplane under consideration when set at an angle of 25° to 35° and that split flaps cause a slight improvement when set at 15° to 25°.

When slotted flaps are set at 40° to 60° the profile drag is materially increased and the maximum lift coefficient is also reached. This increase of drag has been found sufficiently effective to steepen the gliding angle without giving the feeling of "dropping like a brick."

The author is indebted to Mr. R. S. Stafford and Mr. G. C. Russell for their assistance in preparing this report and to Messrs. Handley Page for permission to publish certain information obtained in their own research departments.

A Method for Determining the Ultimate Strength of a Column Swaged at the Ends

J. M. GWINN, JR. AND ROY A. MILLER, *Consolidated Aircraft Corporation*

(Received February 13, 1935)

IT is sometimes necessary to swage an end of a strut, as shown in Fig. 1. This operation may reduce the column strength a considerable amount below that of the unswaged strut. The following derivation yields a method of determining the ultimate strength of such a strut.

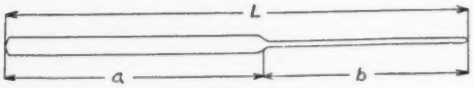


FIG. 1.

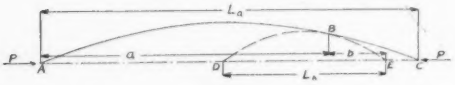


FIG. 2.

Let *ABC*, Fig. 2, be a pin ended strut of length L_a deflected under its Euler load "*P*." Let *DBE* be a pin ended strut with a smaller moment of inertia, a shorter length L_b, but with the same ultimate column strength, deflected under its Euler load "*P*." It is assumed that the ends of the two struts lie on the straight line which is the line of action of the force "*P*" and that they are so deflected by this force that their elastic axes are tangent at *B*.

On page 564, Vol. II, of *Strength of Materials* by Timoshenko, it is shown that at any point the deflection of a strut fixed at one end and with a sufficiently large axial load applied at the other end is given by the equation $y=\delta(1-\cos Kx)$. The origin for "*y*" and "*x*" is at the fixed end and "δ" is the maximum deflection which occurs at the free end. For convenience the deflection at any point may be measured from the line of action of the force "*P*" which passes through the free end and is parallel to the "*x*" axis: $\delta-y=\delta\cos Kx$. It is evident that the fixed ended strut may be considered to be half of a symmetrical pin ended strut and that the value, $\delta\cos Kx$, yields the ordinates to the elastic curve of a pin ended strut when the origin is at the intersection of a line drawn through the ends and the plane of symmetry. If the origin is at the end of the strut, the value of the ordinate to the elastic curve at any point measured from the line of action of the force "*P*" through the ends becomes "$\delta\sin Kx$."

At "*B*," Fig. 2, we have the deflection equal to

$$\delta_a \sin aK_a=\delta_b \sin bK_b. \tag{1}$$

The slope equation equals the differential of the elastic curve with respect to *x* and equals:

$$i=\delta K\cos Kx.$$

At "*B*," Fig. 2, we have the slope equal to

$$\delta_a K_a\cos aK_a=-\delta_b K_b\cos bK_b. \tag{2}$$

Dividing Eq. (1) by Eq. (2) we have

$$\frac{1}{K_a}\tan aK_a=-\frac{1}{K_b}\tan bK_b. \tag{3}$$

As *K* equals $(P/EI)^{\frac{1}{2}}$ and *J* equals $(EI/P)^{\frac{1}{2}}$ Eq. (3) may be written $J_a\tan a/J_a=-J_b\tan b/J_b$. This equation may be simplified to the following:

$$\left(\frac{I_a}{I_b}\right)^{\frac{1}{2}}\tan\frac{a}{J_a}=-\tan\frac{b}{J_b}. \tag{4}$$

Eq. (4) holds when the values of J_a and J_b are computed for the Euler load "*P*," this Euler load being the ultimate column strength of the struts *ABC*, *DBE* and *ABE*.

If the strut is swaged an equal amount and an equal length at each end, the solution is obtained as follows:

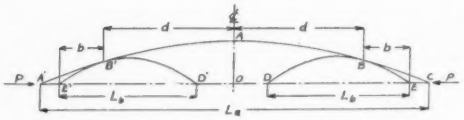


FIG. 3.

The struts in Fig. 3 are similar to those in Fig. 2 except that the strut DBE appears also at the left side of the sketch as $D'B'E'$. It is assumed that $D'B'E'$ has the same length and moment of inertia as DBE and that its elastic axis is tangent to the elastic axis of strut $A'B'ABC$ at B'. B' and B are at equal distances "d" from the plane of symmetry. The three struts are assumed to have the same Euler critical load, as was assumed with the two struts in the first derivation.

If the origin is at "0," the distance from the line of action of the force "P" to any point on the elastic axis of strut ABC is:

$$y=\delta_a \cos xK_a. \tag{5}$$

The slope at any point equals

$$i=\frac{dy}{dx}=-\delta_a K_a \sin xK_a. \tag{6}$$

If the origin is at D, the distance from the line of action of the force "P" to any point on the elastic axis of strut DBE is:

$$y=\delta_b \sin xK_b. \tag{7}$$

The slope at any point equals

$$i=\frac{dy}{dx}=\delta_b K_b \cos xK_b. \tag{8}$$

At B:

$$y=\delta_a \cos dK_a=\delta_b \sin bK_b, \tag{9}$$

$$i=\delta_a K_a \sin dK_a=\delta_b K_b \cos bK_b. \tag{10}$$

Dividing Eq. (9) by Eq. (10):

$$\frac{1}{K_a}\cot dK_a=\frac{1}{K_b}\tan bK_b. \tag{11}$$

This equation may be written:

$$\left(\frac{I_a}{I_b}\right)^{\frac{1}{4}}\cot\frac{d}{J_a}=\tan\frac{b}{J_b}. \tag{12}$$

Eq. (12) holds when the values of J_a and J_b are computed for the Euler load "P," this Euler load being the column strength of the struts $A'AC$, DBE and $E'B'ABE$.

To check these formulas, a round duralumin tube was selected, cut to length and the ends carefully squared. It measured $1.253\times.051$ and its length was 44.97 inches when mounted between knife edges in the testing machine.

$$A=.1926 \text{ ins.}^2$$

$$I=.03484 \text{ ins.}^4,$$

$$\rho=.425,$$

$$\frac{1}{\rho}=\frac{44.97}{.425}=105.7.$$

This tube was mounted on knife edges in an 80,000 lb. testing machine and failed under a compression load of 1804 lbs. The modulus of elasticity, calculated from the Euler formula is

$$E=\frac{1804\times44.97^2}{3.1416^2\times.03484}=10{,}610{,}000 \text{ lbs./sq. in.}$$

This tube was then swaged at one end and the swaged end cut and squared so that the length remained the same. The dimensions of the swaged end were found to be $.756''\times.078''$. In the swaging operation, the taper from the 1.253" diameter to the .756" diameter extended over a distance of $3\frac{19}{32}$ inches and was uniform. The length "b" (Fig. 1) was taken as the distance from the end of the strut to the mid-point of the tapered portion and equaled 11.985 inches. The length "a" equaled $44.97-11.985=32.985$ inches.

$$A=.16614 \text{ ins.}^2,$$

$$I=.00967 \text{ ins.}^4,$$

$$\left(\frac{I_a}{I_b}\right)^{\frac{1}{4}}=\left(\frac{.03484}{.00967}\right)^{\frac{1}{4}}=1.89813.$$

Let $P=1312$ lbs.

$$J_a=\left(\frac{10{,}610{,}000\times.03484}{1312}\right)^{\frac{1}{4}}=16.78533,$$

$$J_b=\left(\frac{10{,}610{,}000\times.00967}{1312}\right)^{\frac{1}{4}}=8.84309,$$

$$1.89813\tan\frac{32.985}{16.78533}=-\tan\frac{11.985}{8.84309},$$

$$4.56172=4.56823.$$

The difference is so small that the assumed load, 1312 lbs., satisfies the equation to the nearest pound. This strut was mounted on knife edges in the 80,000 lb. testing machine and failed under a compression load of 1329 lbs.

The tube was then swaged at the large end so that both ends were alike. The ends were cut and squared so that the length was $45\frac{1}{32}$ inches and "b" equaled 11.985 inches at each end. Then $2d=45.3125-(2\times11.985)=21.3425$ inches. When measured, the second swaged end was .76"×.080" and $I=.01001$. It was decided to use an average "I" for the smaller ends: $(.00967+.01001)/2=.00984$ ins.[4]

$$\left(\frac{I_a}{I_b}\right)^{\frac{1}{3}}=\left(\frac{.03484}{.00984}\right)^{\frac{1}{3}}=1.88166.$$

Let $P=1106$ lbs.

$$J_a=\left(\frac{10{,}610{,}000\times.03484}{1106}\right)^{\frac{1}{3}}=18.28181,$$

$$J_b=\left(\frac{10{,}610{,}000\times.00984}{1106}\right)^{\frac{1}{3}}=9.71578,$$

$$1.88166\cot\frac{10.67125}{18.28181}=\tan\frac{11.985}{9.71578},$$

$$2.84893=2.85198.$$

This difference is so small that the assumed load, 1106 lbs., satisfies the equation. This strut was mounted on knife edges in the 80,000 lb. testing machine and failed under a compression load of 1123 lbs.

The Computation of the Apparent Mass of Dirigibles

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(Received November 6, 1934)

NUMERICAL information about the apparent mass of dirigible hulls is of practical value in airship design, chiefly in connection with the computation of the structural loads during various maneuvers. So far, the magnitude of the apparent mass seems to have been numerically determined for ellipsoids only.[1] There has further been proposed an approximate formula for hull shapes with a middle body closely approaching a circular cylinder, based on the apparent mass of a "semi-body."[2]

While these data go a far way towards satisfying the present needs of the designer, still, niceties of technical specifications or possibly actual advances of airship design in the future may make it desirable to have a general method available whereby the value of the apparent mass of any airship hull can be numerically determined. Such a method will be discussed in what immediately follows, and the method will be outlined by way of applying it at once to particular examples.

These dirigible shapes, presently to be investigated, are not defined by their own geometric properties, like ellipsoids, but they are the result of certain geometric conditions imposed on the flow to which they give rise. In other words, a shape is determined matching a flow, and not a flow matching a shape. Such an indirect way of solving the flow problem is indeed easier than the direct way if it comes to the use of numerical methods. The shapes that may thus be obtained are of enough variety for all practical purposes. This method of fitting the shape to the flow has already been used by Rankine for the study of two-dimensional flow problems.[3]

It has again been used for the study of three-dimensional flow problems in connection with a study of dirigible air forces by Fuhrmann.[4] Fuhrmann imposed suitable conditions on the flow, and computed therefrom the fluid motion and the solid shape necessary to produce that flow by its motion. From the velocity distribution so determined he computed the pressure distribution over the solid surface, and compared it with the actual pressure distribution measured by a windtunnel test. He found moderately good agreement and so justified the application of theoretical hydrodynamics to aeronautical flow problems, which has opened up then unaccessible regions of aerodynamics to a remarkable degree. Fuhrmann's paper certainly is a landmark in the literature of aerodynamics.

Proceeding now to use some of Fuhrmann's numerical data computed by him in connection with certain of his airship shapes for the computation of the apparent mass of the latter, for Fuhrmann does not give the values of the apparent mass. On the contrary, his paper is surprisingly meager concerning the data necessary for their computation. The author computes the apparent mass from the distribution of the fictitious sources and sinks giving rise to the fluid motion. The apparent mass is equal to the product of the density of the fluid and the static moment of those sources and sinks, assuming the specific density of the hull to be equal to that of the surrounding fluid.[5] Hence, this static moment divided by the hull volume gives the factor of apparent mass, and the excess of this factor over ONE gives the factor of additional apparent mass, a non-dimensional quantity depending on the proportions of the shape only.

Fuhrmann's airship shape IV (see Fig. 2) has been treated in the fullest and most complete way. Its volume is stated to be .0182 cu.m, its length 1145 mm, its maximum diameter 188 mm.

[1] H. Lamb, *Hydrodynamics*, 5th ed., p. 146, Cambridge, 1924.

[2] Max M. Munk, *The Drag of Zeppelin Airships*, T. R. 117, N. A. C. A., 1921.

[3] W. J. M. Rankine, *On Plane Water Lines . . . etc.*, p. 369, Phil. Transactions of the Royal Society of London, 1864.

[4] Georg Fuhrmann, *Theoretische und experimentelle Untersuchungen an Ballonmodellen*, Vol. 5, Jahrb. der Motorluftschiffahrt Studien Ges., 1911–12.

[5] W. F. Durand, *Aerodynamic Theory*, Vol. 1, p. 258, Julius Springer, Berlin, 1934.

The source intensity, arranged according to Fig. 1, extends through a portion of the airship axis of a length equal to three times 38 cm. We are at last informed that if for a geometrically similar flow the source intensity would have extended through three times 10 cm only, a quantity $a/2c$ would have been equal to 30, which is an inverse area and measured in units of $1/dm^2$. In the above expression a denotes the velocity of flight, and $4\pi c$ denotes the volumetric delivery rate per unit time of all positive sources or of all negative sources. Fig. 1 shows that the distance between the centers of gravity of all sources and of all sinks is equal to one-sixth of a meter. From all of which the static moment of the source intensity follows to be $.16667 \times 2\pi/(a/2c)$. The volume of the solid corresponding to this source distribution is equal to the model volume divided by the cube of the reduction scale 3.8, giving a volume equal to $18.2/3.8^3$. The factor of apparent mass results therefore

$$k=\frac{2\times\pi\times 1.6667\times 3.8^3}{30\times 18.2}=1.0438.$$

This, by subtracting 1, gives a factor of additional apparent mass equal to 4.4 percent.

Proceeding to compute in a similar way this factor for the shapes III, V and VI (see Fig. 2),

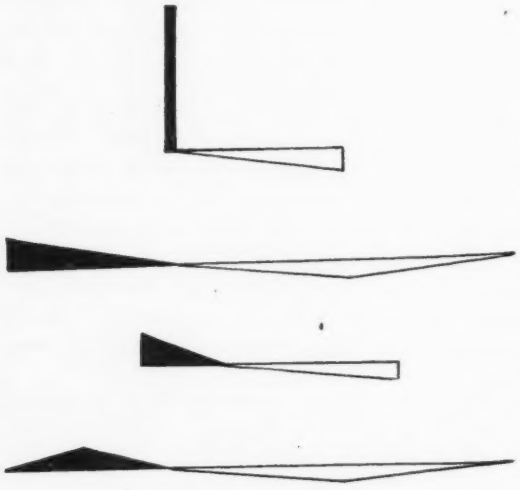

FIG. 1. Diagrams III, IV, V and VI. (Top to Bottom.)

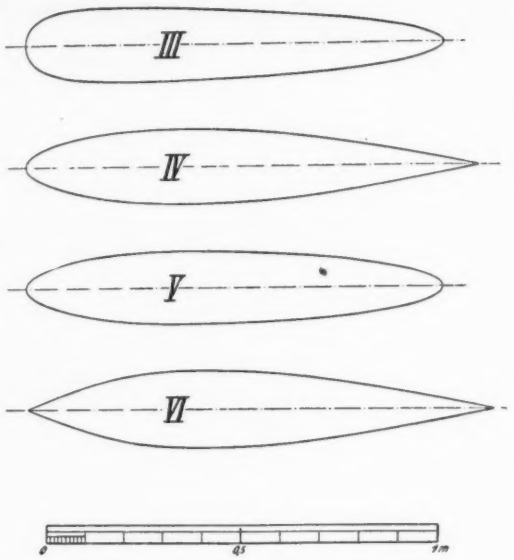

FIG. 2. Airship Shapes.

this cannot be done as accurately as with shape IV, because Fuhrmann's paper is deficient in one quantity necessary for the computation. He fails to give the intensity of the source distribution, or the same divided by the velocity of motion, for the volume of his model; or otherwise, he fails to give this volume for the distribution intensity reported. Fuhrmann only provides information about the type of the source distribution as such, but with respect to its numerical intensity only indirect ways of finding out are left. With shape VI this difficulty is more apparent than real, because there the length of the axis through which the sources and sinks are distributed is practically equal to the length of the balloon. At both ends, at the bow and stern, the intensity gradually fades out towards the end to zero. The quantity $a/2c$ refers to a length of 3, and the length of the model is given in Table X of Fuhrmann's paper ($L=1186$ mm). There is therefore no real obstacle to the computation of the longitudinal apparent mass. This computation nevertheless fails at first, giving a negative additional apparent mass, which is impossible. A study of Fuhrmann's figures reveals that there seems to be a misprint in the given value of $a/2c$, a "9" seems to be printed instead of a "0."

39.51 should read 30.51, I believe. The reported value would indeed give too small a ratio of the maximum diameter to the length. With the above correction inserted, the factor of additional apparent mass becomes 4.8 percent.

With models III and V, the problem arises to determine the ratio of the hull length to the portion of the axis occupied by sources and sinks. With III, it has been assumed by way of approximation that the sinks extend up to the tail end, and that the distance between the bow and the concentrated source is the same as if there were no sinks but that distance generated by the point source only. This distance is then $(c/a)^{\frac{1}{2}}$, where these letters have the same meaning as above. The sinks being distributed with linearly varying intensity through the length 1, the distance between their center of gravity and the point source is 2/3. The shape corresponding to this distribution and to the reported value of $2a/c$ is a reduction of the model in the ratio 1 to 10.06. That completes the data needed, and the factor of additional apparent mass becomes 4.6 percent.

With model V, it has at last been assumed that the sources and sinks extend up to the focus of the half-ellipses with one and two-thirds of the hull length as major semi-axes, and with the maximum diameter as minor semi-axes. The generating source and sink intensity really giving rise to an ellipsoid, is of a linearly distributed type as in this case. This assumption gives an enlargement scale of exactly 7, and a factor of apparent additional mass of about 9 percent. The assumption is not exact enough for determining this factor more closely. The results are merely larger than with the other investigated models, obviously because both nose and stern are blunt.

All shapes in Fuhrmann's paper have the same ratio between the surface and the 2/3 power of the volume. The above computations seem to indicate that under such circumstances the apparent mass does not vary widely, and is largely determined by the shape of the ends.

The same method for the determination of the apparent mass of solids of revolution can also be used for lateral motion. Doublets, rather than single sources or sinks, then have to be employed, distributed in suitable intensity along the axis. The lateral apparent additional mass is of course much different in numerical value from the longitudinal mass. This phraseology of dealing with different masses for different directions of motion is somewhat confusing, and even inexact, since the momentum is not necessarily parallel to the motion, and we would have to distinguish again between masses for the different components of the momentum. It would be much better if the vector analytic terms would be adopted, regarding these so-called different masses only as components of one physical quantity of higher complexity, as we are long accustomed to do with the stress. There, the different normal stresses and shear stresses are generally regarded as portions of one state of stress, and the geometry of the state of stress and of the relation between momentum and mass of a solid moving through a perfect fluid is identical. In either case we have a symmetric dyadic.

A remark on the reality of the physical momentum of the fluid motion related to the velocity of the solid by said dyadic may be made in closing. This momentum cannot be computed from the fluid motion by an ordinary space integral over the momenta of all particles. The momentum is not concentrated in the vicinity of the moving solid, such as the kinetic energy is, and the value of the integral depends on the shape of the infinite boundary surface up to which it is extended. This now is not at all in contradiction to the physical reality of the momentum, but merely illustrates the true state of facts. The space integral is equal by geometric rule to the surface integral of the potential, (generalized Gauss theorem), both integrals expressing the summation of vectors, giving again a vector as result. The surface integral, aside from a certain constant factor, taken over the surface of the solid, gives directly the impulse imparted by the solid to the fluid while the motion was created. The surface integral over the outer boundary may be zero, and can always be obtained as zero by suitably modifying the conditions at infinity, namely by approaching the infinite flow by enlarging a finite quantity of fluid surrounded by a vacuum in the midst of which the solid is located. In that case, the

momentum under discussion is exactly equal to the momentum of the fluid, taken in its ordinary physical meaning.

In other cases, it may not be, because the portion of the fluid considered may have transmitted momentum to other masses outside, either other fluid masses not included in the integral, or to solids or a solid shell surrounding the fluid. In any case, the momentum under discussion is still equal to the momentum in the system of all masses constituting a complete isolated dynamic system. In other words, the momentum is physically real, and constitutes the momentum of the complete system, namely of all masses, in contradistinction to the mass of a certain portion of the fluid being included artlessly in the process of integration. This is a fine point, and it may appear purely academical, but it is far from it. The question whether the flow momentum is a formal or a physically real quantity is of practical significance. The realization that this momentum is of physical reality makes it certain once for all that the flow momentum complies certainly and always with all laws of mechanics relating to momenta. A quantity merely in the nature of and formally analogous to a momentum may follow only a portion of these laws, or follow them only up to a certain point; a mere analogy may not extend all the way through.

The discussion shows also that the momentum, and hence the apparent mass, can be directly computed from the potential, which is often simpler than using the kinetic energy, which is the conventional method. By way of example, take the translational motion of a sphere with the velocity of translation U. Its potential is $\frac{1}{2}\cos\theta Ua^3/r^2$; r and θ being the polar coordinates, and a the radius of the sphere. At the sphere's surface, $r=a$, and hence this potential becomes $\frac{1}{2}Ua\cos\theta$. This, multiplied by the density of the fluid, constitutes the impulsive pressure creating the flow, and is seen to be equal to a buoyant pressure in a fluid having a specific gravity equal to *half* the density of the fluid. Hence the buoyancy would be *half* of the weight of the fluid filling the sphere, and hence the factor of apparent additional mass is seen equal to ONE-HALF. This agrees of course with the value found in the conventional way by using the kinetic energy.

Post Scriptum. The question of the flow momentum can be disposed of in a still clearer way by means of the following two propositions, *viz.*: (a) The apparent momentum of the outside flow is equal to the negative momentum of the fictitious inside flow obtained by continuing the outside flow free of rotation, but not free of divergence. (b) The momentum of any irrotational flow of a fluid of unit density within a closed surface is equal to the negative static moment of its source and sink system.

Dynamic Longitudinal Stability Charts for Design Use

L. E. ROOT, *Douglas Aircraft Company*

(Received November 1, 1934)

THROUGH the pioneering efforts of such men as Bryan, Routh, Lanchester, Hunsaker, von Kármán, Trefftz, Fuchs, Hopf, Glauert, and Gates, fundamental dynamic longitudinal stability theory has been well developed. However, those who have studied the theory realize its inherent complication so far as any rapid application to design is concerned. The first serious attempt to simplify the application of the theory by the use of stability diagrams was made by von Kármán and Trefftz[1] in 1913. Later, in 1927, Gates published an English paper[2] which aptly illustrated the practical application of classical dynamic stability theory to design.

The present paper is a modification of Gates' previous work. The same method of attack is followed, but American conventions are used throughout. The stability diagrams, by the use of more modern basic assumptions, are brought up to date as nearly as possible in order to be more applicable to present day airplanes. By the use of the dimensionless method[3] of representing important factors appearing in the theory, the problem of application of the theory is simplified and comparison between dissimilar planes is much facilitated.

It is evident that a presentation of the complete development of longitudinal dynamic stability theory in a paper of this length would be impossible. Since there are publications[4-5] in which the theory is given in detail, a knowledge of the basic principles will be assumed throughout the present discussion and the subject will be treated more from an engineering point of view.

The trend toward increasing refinement in modern airplane design, especially in flying characteristics, justifies the introduction of a concise method for determining the degree of dynamic stability. In the modern transport airplane, smooth flight, which is necessary for the use of automatic piloting devices, and the comfort of both passengers and pilot, requires a high degree of dynamic longitudinal stability.

THEORETICAL DISCUSSION

By employing the non-dimensional system developed by Glauert,[3] using wind axes, the stability quartic of the longitudinal dynamic motion may be expressed as follows:

$$\begin{vmatrix} \lambda+x_u & x_w & \mu\dfrac{C_L}{2} \\ z_u & \lambda+z_w & \mu\left[\dfrac{C_L}{2}\tan\theta-\lambda\right] \\ m_u & m_w & \lambda^2+m_q\lambda \end{vmatrix} = 0. \quad (1)$$

The coefficients in this equation are discussed in the following paragraphs.

Eq. (1) may be reduced to the form:

$$\lambda^4+B\lambda^3+C\lambda^2+D\lambda+E=0, \quad (2)$$

where the coefficients B, C, D, and E are functions of the various stability derivatives; namely,

$$B=z_w+m_q+x_u,$$

$$C=m_q(z_w+x_u)+\mu m_w+x_u z_w-x_w z_u,$$

$$D=m_q(x_u z_w-x_w z_u)+\mu m_w\left(x_u-\frac{C_L}{2}\tan\theta\right)-\mu m_u\left(\frac{C_L}{2}+x_w\right),$$

$$E=\frac{C_L}{2}\mu m_w(z_u-x_u\tan\theta)-\frac{C_L}{2}\mu m_u(z_w-x_w\tan\theta).$$

[1] von Kármán und Trefftz, *Über Längsstabilität und Längsschwingugen von Flugzeugen*, W. G. L. Jahrb., Kriegsjahr 1914/15.

[2] Gates, S. B., *A Survey of Longitudinal Stability Below the Stall with an Abstract for Designer's Use*, British A. C. A., R. & M. 1118, July, 1927.

[3] Glauert, H., *A Non-dimensional Form of the Stability Equations of an Airplane*, British A. C. A., R. & M. 1093, March, 1927.

[4] Bairstow, *Applied Aerodynamics*, London, 1920, Chap. 10.

[5] Wilson, E. B., *Aeronautics*, Chap. VIII, Wiley and Sons, 1920.

For gliding flight below the stall of the airplane, all of the derivatives in the above non-dimensional method are positive except m_w, m_u, and x_w. For a proper understanding of the application of the stability theory, it is well to explain the meaning of the various "stability derivatives." Fig. 1 illustrates the conventions used. The forces along the various axes are X, Y and Z, and the linear velocities are u, v and w. The three angular directions are designated by ϕ, θ and ψ while p, q and r represent the angular velocities.

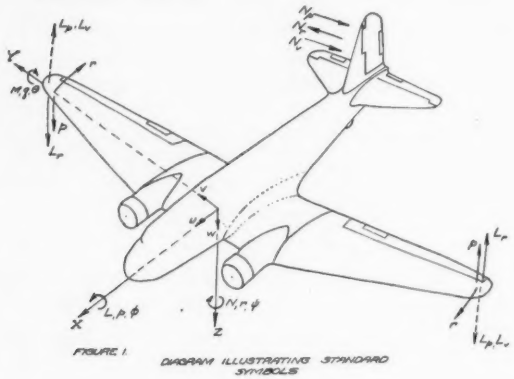

FIGURE 1. DIAGRAM ILLUSTRATING STANDARD SYMBOLS

There are seven different non-dimensional stability derivatives appearing in the theory. They are as follows:

(1) x_u, representing the force along the X axis due to a forward linear velocity. (In non-dimensional units, x_u is taken equal to the drag coefficient, C_D.)

(2) x_w, the force along the X axis due to a downward linear velocity, w.

(3) z_w, the force along the Z axis due to a downward velocity, w.

(4) z_u, representing the force along the Z axis due to a forward linear velocity, u. (In non-dimensional units, z_u is taken equal to the lift coefficient, C_L.)

(5) m_w, the pitching moment due to a downward linear velocity, w. (m_w corresponds to the static stability of the airplane.)

(6) m_q, the pitching moment due to an angular velocity of pitch.

(7) m_u, the pitching moment due to a forward velocity, u. (In gliding flight, m_u is assumed equal to zero.)

In the non-dimensional system x_w is expressed as $\frac{1}{2}[(dC_D/d\alpha)-C_L]$. If we now take $dC_D/d\alpha = (dC_D/dC_L)(dC_L/d\alpha)$ and assume the drag coefficient to be of the form $C_D=C_{D_P}+C_{D_i}$, where C_{D_P} is the parasite drag coefficient and C_{D_i} is the induced drag coefficient, then it is possible to express x_w in the following form:

$$x_w=\tfrac{1}{2}\left[\frac{d(C_{D_P}+C_{D_i})}{dC_L}\frac{dC_L}{d\alpha}-C_L\right].$$

If we assume an elliptical lift distribution then C_{D_i} may be taken as C_L^2/π A.R. where A.R. is the aspect ratio of the wing. We then have $dC_D/dC_L=2C_L/\pi$ A.R., from which

$$x_w=\tfrac{1}{2}\left[\frac{2C_L}{\pi \text{ A.R.}}\left(\frac{k}{1+(k/\pi \text{ A.R.})}\right)-C_L\right]$$

$$=-\tfrac{1}{2}\left[\frac{1-(k/\pi \text{ A.R.})}{1+(k/\pi \text{ A.R.})}\right]C_L,$$

where k is the slope of the lift curve for infinite aspect ratio.

The stability derivative, z_w, is taken equal to $\frac{1}{2}[(dC_L/d\alpha)+C_D]$. Since $dC_L/d\alpha$ may be equated to $k/[1+(k/\pi \text{ A.R.})]$ we have $z_w=\frac{1}{2}\{k/[1+(k/\pi \text{ A.R.})]+C_D\}$. In the calculations, C_D is neglected in the above expression for z_w since it is small in comparison with $dC_L/d\alpha$.

The "static stability" derivative, m_w, is expressed as $-\frac{1}{2}(l^2/k_B^2)(dC_m/d\alpha)$ or $-\frac{1}{2}(l^2/k_B^2)\times(dC_L/d\alpha)(dC_m/dC_L)$, where k_B is the radius of gyration about the lateral axis of the airplane. It is noted that dC_m/dC_L is the slope of the static pitching moment curve.

In the non-dimensional system, m_q, the damping in pitch due to a pitching velocity, is taken as $C\frac{1}{2}(S_t/S)(l^2/k_B^2)(dC_{L_t}/d\alpha_t)$, where S_t is the area and $dC_{L_t}/d\alpha_t$ is the slope of the lift curve of the horizontal tail surfaces of the airplane. If we assume that all of the pitching moment arises from the horizontal tail surfaces alone, C is taken equal to unity. However, the fuselage usually contributes some damping, giving C values ranging from 1.0 to 1.3. The constant, C, must be estimated, keeping in mind the relative damping effects of various fuselage shapes. A good average value to use is $C=1.2$ in case there are no available comparisons.

Longitudinal dynamic stability may be completely described in terms of the above derivatives, C_L, and the variable "μ," which always appears in combination with m_w or m_u. The parameter "μ" is called the "relative density" and is equal to the following:

$$\mu=\frac{M}{\rho Sl}=\frac{W}{\rho glS}=\frac{l_w}{\rho gl}=13.08\frac{l_w}{\sigma l},$$

where

M = Mass of the airplane in slugs,
W = Weight of the airplane in pounds,
S = Wing area in square feet,
$l_w = W/S$ = Wing loading in pounds per square foot,
ρ = Air density in slugs per cubic foot,
σ = Relative air density,
l = Length in feet from the center of gravity of the airplane to the center of pressure of the horizontal surfaces.

It is logical to call μ the "relative density" since it is seen to be the mass of the airplane divided by the mass of a volume of air upon the wing surface with the height "l." The "relative density" proves useful in concisely summarizing several important parameters, since any variation in linear scale, wing loading, or density is included in "μ." The unit of time in the dimensionless system is chosen as $T=M/\rho SU$ where U is the velocity of flight. The unit of length is arbitrarily chosen as l, consequently the unit of velocity becomes l/T or U/μ.

Graphical Representation of Stability

From the stability theory, it is known that such a system as that represented by Eq. (2) may, by the use of Bairstow's "approximate factorization," be reduced to

$$[\lambda^2+B\lambda+C]\left[\lambda^2+\left(\frac{D}{C}-\frac{BE}{C^2}\right)\lambda+\frac{E}{C}\right]=0. \quad (3)$$

The first quadratic, the so-called "short oscillation," has negligible influence on longitudinal stability since the period is short and the oscillation is heavily damped. The second quadratic, however, represents the "phugoid" motion, a slightly damped periodic mode which directly determines the dynamic longitudinal stability of an airplane. It is this motion with which we are most concerned.

If all the coefficients B, C, D and E of the equation $\lambda^4+B\lambda^3+C\lambda^2+D\lambda+E=0$ are positive and if Routh's discriminant, $R=BCD-D^2-B^2E$ is greater than zero, then the system represented by the above equation is said to be stable. It happens that E is the critical coefficient such that when it equals zero, the system becomes unstable and the flight path becomes divergent. Since E, as mentioned previously, may be expressed in terms of the various stability derivatives, it may be plotted after a suitable choice of coordinates is made. If we let $X=\mu m_w$ and $Y=m_q$ remembering that $m_u=0$ for gliding flight, we have $E=(X/2)(C_Lz_u-x_uC_D)$. When $E=0$ we see that one stability boundary will be given by the Y axis. (Since $CX=0$.) Knowing the values of the coefficients B, C, D and E in terms of the stability derivatives it is possible to obtain $R=BCD-D^2-B^2E$ in terms of the coordinates, and plot the curve $R=0$ to give the other boundary of stability beyond which the state of motion is one of increasing oscillations. It is convenient, since D^2 is always relatively small, to use the expression $R=B(CD-BE)$ for plotting. When $R=0$ we have $CD-BE=0$ which is identical to the expression for $K=0$ since $K=(DC-BE)/C$. (K is the damping factor in non-dimensional units.) Thus, the curve $K=0$ gives a very close approximation to the other stability boundary. If we now plot families of constant period and damping factor curves using the coordinates X and Y, we have a complete picture of the phugoid motion of a given airplane represented by a point on the graph.

The period and damping factor families in the non-dimensional system are represented by the following equations:

(a) $$2\pi/p=\left[\frac{E}{C}-\frac{1}{4}\left(\frac{D}{C}-\frac{BE}{C^2}\right)^2\right]^{\frac{1}{2}}=\text{a constant},$$

(b) $$K=\frac{D}{C}-\frac{BE}{C^2}=\text{a constant},$$

which define the roots of the second quadratic in Eq. (3).

Because we wish to have the period and damping factor in the non-dimensional system converted into the English system, we multiply by the appropriate factors:

$$P \text{ (seconds)} = Tp; \quad \text{Damping Factor} = K/2T.$$

Making use of the condition, $W=\rho(U^2/2)SC_L$ from which $U=(2l_w/\rho C_L)^{\frac{1}{2}}$, we have as the value of T the following:

$$T=\frac{\dot{M}}{\rho SU}=\frac{Mg}{\rho gSU}=\frac{l_w}{\rho g(2l_w/\rho C_L)^{\frac{1}{2}}}=\frac{1}{(2\rho_0)^{\frac{1}{2}}}\frac{1}{g}\left(C_L\frac{l_w}{\sigma}\right)^{\frac{1}{2}}.$$

Substituting for T we have,

$$P=p\left[\frac{(C_L)^{\frac{1}{2}}}{(2\rho_0)^{\frac{1}{2}}}\frac{1}{g}\right]\left(\frac{l_w}{\sigma}\right)^{\frac{1}{2}}$$

and

$$\text{D.F.}=K\left[\frac{1}{2}\frac{(2\rho_0)^{\frac{1}{2}}g}{(C_L)^{\frac{1}{2}}}\right]\left(\frac{\sigma}{l_w}\right)^{\frac{1}{2}}.$$

By varying the lift coefficient, C_L, we get the following set of factors to be used in converting the period and damping factor into the appropriate units:

C_L	$[1/(2\rho_0)^{\frac{1}{2}}][(C_L)^{\frac{1}{2}}/g]$	$\frac{1}{2}\{[1/(2\rho_0)^{\frac{1}{2}}][(C_L)^{\frac{1}{2}}/g]\}^{-1}$
0.2	0.201	2.485
0.6	0.349	1.433
1.0	0.451	1.108
1.4	0.533	0.938

To get the time to damp to one-half amplitude, we use the relation:

$$T_{\frac{1}{2}}=\frac{\log_e 2}{\text{D.F.}}=\frac{0.693}{\text{D.F.}}.$$

As previously explained it has been found convenient to take as X and Y, the following:

$$X=\mu m_w=-\mu\frac{1}{2}\frac{l^2}{k_B{}^2}\frac{t}{l}\frac{dC_L}{d\alpha}\frac{dC_M}{dC_L},$$

$$Y=m_q=C\frac{1}{2}\frac{S_t}{S}\frac{l^2}{k_B{}^2}\frac{dC_{L_t}}{d\alpha_t}.$$

From the discussion of the various derivatives, it is seen that X is proportional to the relative density, μ, and the damping in pitch due to a downward linear velocity, or essentially the static longitudinal stability of the airplane; while Y is proportional to the damping in pitch due to an angular velocity of pitch, caused largely by the horizontal surfaces. The above choice for X and Y is very convenient since the slopes, $dC_L/d\alpha$ and dC_M/dC_L, are known from wind tunnel results, and the slope $dC_{L_t}/d\alpha_t$ can easily be calculated. In the procedure of any serious design, all of the appearing quantities will be known with the possible exception of k_B, the radius of gyration about the lateral axis. However, the moment of inertia may be calculated without great difficulty using a balance diagram for the airplane. From the moment of inertia, B, and weight, W, k_B is simply $(B/W)^{\frac{1}{2}}$.

Discussion of Parameters

It will be noticed that all of the variables appearing in the several stability derivatives do not occur in the chosen coordinates X and Y. For this reason it is necessary to make assumptions regarding constant values of the extraneous variables $dC_L/d\alpha$, the slope of the lift curve, and C_D, the drag coefficient, in order to be able to represent the period and damping factor in graphic form.

From a study of flight test data and wind tunnel tests on various airplanes, it was decided to plot stability diagrams representing two groups of airplanes; those corresponding to clean, modern airplanes with highly efficient wing sections representing an upper limit, and those corresponding to the older type with higher parasitic resistance and less efficient airfoil sections, representing a lower limit. For the first case, examination of wind tunnel tests on several modern airplanes indicated an optimistic upper limit for $dC_L/d\alpha$ of 4.8. If we assume that the formula, $dC_L/d\alpha=k/(1+k/\pi \text{ A.R.})$ where A.R. is the aspect ratio of the wing and k is the slope of the lift curve for infinite aspect ratio, holds for a wing having a lift distribution closely approximating the elliptic, we have as the values of A.R. and k corresponding to $dC_L/d\alpha=4.8$, the following: A.R.$=7.7$, $k=6.0$. For the second case we have as a representative value, $dC_L/d\alpha=4.0$. Here the corresponding values of A.R. and k are: A.R.$=4.7$, $k=5.5$. Previously the value $k=5.5$ for the slope of the lift curve corrected to infinite aspect ratio has been taken

as standard. However, an investigation of the slopes of various lift curves in recent wind tunnel tests indicates a higher value, and $k=6.0$ has been chosen to represent the optimum upper limit. Of course the theoretical lift curve slope for infinite aspect ratio is 2π or 6.28.

We assume that the drag coefficient has the form,

$$C_D = f/S + C_L^2/\pi e \text{ A.R.},$$

where $f/S = C_{D_P}$ = parasite drag coefficient,
$C_L^2/\pi e$ A.R. $= C_{D_i}$ = induced drag coefficient,
f = equivalent parasite area,
S = wing area,
e = airplane efficiency factor,
A.R. = aspect ratio of the wing.

A good value of "e" based on wind tunnel results and flight tests seems to be 0.9. The formula, $C_{D_i} = C_L^2/\pi$ A.R. assumes an elliptical lift distribution. The factor "e" in the above formula for C_D corrects for non-elliptical lift distribution and the variation of parasite drag with lift coefficient. Taking $e=0.9$ and using the aspect ratios consistent with the assumption for values of C_{D_i}, we have as representative values of C_{D_i} for the two cases, the following:

$$\text{Case I} \quad C_{D_i} = \frac{C_L^2}{\pi(0.9)(7.7)} = 0.046 C_L^2,$$

$$\text{Case II} \quad C_{D_i} = \frac{C_L^2}{\pi(0.9)(4.7)} = 0.075 C_L^2.$$

A study of available wind tunnel test data shows that appropriate values of C_{D_P} for the two cases are 0.02 and 0.05. The corresponding total drag coefficients are as follows:

$$\text{Case I} \quad C_D = 0.02 + 0.046 C_L^2,$$
$$\text{Case II} \quad C_D = 0.05 + 0.075 C_L^2.$$

It is realized that these two representative cases do not cover all possibilities of combinations of C_D, $dC_L/d\alpha$ and C_{D_P}. For example, for the case in which we have a clean airplane with $C_{D_P}=0.02$, we may have a wing with comparatively low aspect ratio, giving C_{D_i} a value considerably higher than that indicated in Case I. For the case in which C_{D_i} may be low, C_{D_P} may be high. The above assumptions do not cover either of these cases. To satisfy such combinations between C_{D_P} and C_{D_i}, other sets of charts should be plotted combining the high and low values of C_{D_i} and C_{D_P} respectively. [i.e.; for Case III: $C_{D_P}=0.02$, $C_{D_i}=0.075C_L^2$. For Case IV: $C_{D_P}=0.05$, $C_{D_i}=0.046C_L^2$.]

Stability for Gliding Flight

Using the above assumptions for C_D and $dC_L/d\alpha$; we have the following for the values of the stability derivatives in the two cases:

	Case I	Case II
$x_u = C_D =$	$0.02 + 0.046C_L^2$	$0.05 + 0.075C_L^2$
$x_w = -\frac{1}{2}\frac{1-k/\pi \text{ A.R.}}{1+k/\pi \text{ A.R.}} C_L =$	$-0.3C_L$	$-0.2275C_L$
$z_w = \frac{1}{2}\left[\frac{k}{1+k/\pi \text{ A.R.}} + C_D\right] =$	2.4 (C_D neglected)	2.0
$z_u =$	C_L	C_L
$m_w = -\frac{1}{2}\frac{l^2}{k_B^2}\frac{dC_L}{d\alpha}\frac{dC_M}{dC_L} =$	X/μ	X/μ
$m_q = C_{L_t}\frac{1}{2}\frac{S_t}{S}\frac{l^2}{k_B^2}\frac{dC_{L_t}}{d\alpha_t} =$	Y	Y

From these values we can now determine the coefficients B, C, D and E for each case, using different values of the lift coefficient, C_L. Employing a simplified form of equation for the period curves;* namely, $2\pi/p = (E/C)^{\frac{1}{2}}$ = a constant, and the previously chosen coordinates X and Y, we can now plot families of period curves in the non-dimensional system for the two cases. These families are straight lines going approximately through the origin. In the same manner, the constant damping factor families are plotted using the equation

$$K = (D/C) - (BE/C^2) = \text{ a constant.}$$

The two families are represented for different values of C_L in Figs. 2, 3, 4 and 5 for Case I and in Figs. 6 and 7 for Case II. As previously explained the curve $E=0$ establishes one stability

* The more exact form is $2\pi/p = \{(E/C) - \frac{1}{4}[(D/C) - (BE/C^2)]^2\}^{\frac{1}{2}}$. The term $\frac{1}{4}[(D/C) - (BE/C^2)]^2$ is usually very small and may be neglected.

boundary beyond which the motion is divergent and the curve $k=0$ establishes the other boundary, beyond which the motion is one of increasing oscillation. The relations for conversion of the period and damping factors into English units are given below each figure. An example of the use of the charts will be given later.

Conclusions from a Study of the Charts

Many interesting general conclusions can be drawn from a study of the figures for Cases I and II. The first thing noticed is that the region of increasing oscillation expands with increase in lift coefficient, showing that at a high lift coefficient (corresponding to low speed) the

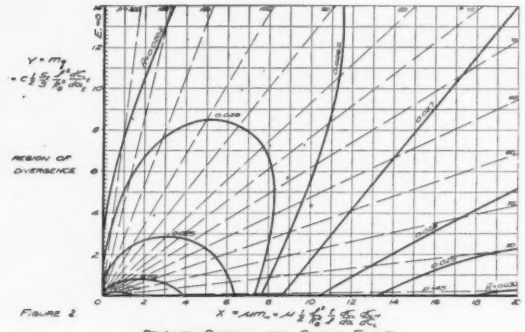

FIGURE 2

STABILITY DIAGRAM FOR GLIDING FLIGHT
CASE I $C_L = 0.2$
DAMPING FACTOR $= 2.48\sqrt{\frac{\rho}{\rho_0}}\,R$ PERIOD $= 0.20\sqrt{\frac{\rho}{\rho_0}}$ P SEC.

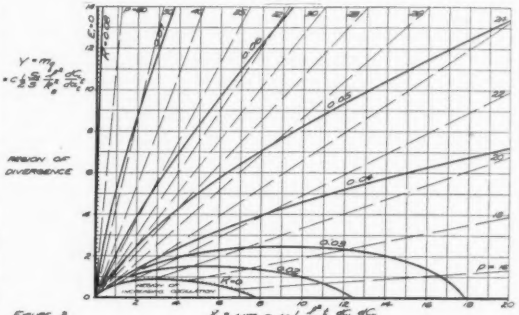

FIGURE 3

STABILITY DIAGRAM FOR GLIDING FLIGHT
CASE I $C_L = 0.6$
DAMPING FACTOR $= 1.48\sqrt{\frac{\rho}{\rho_0}}\,R$ PERIOD $= 0.34\sqrt{\frac{\rho}{\rho_0}}$ P SEC.

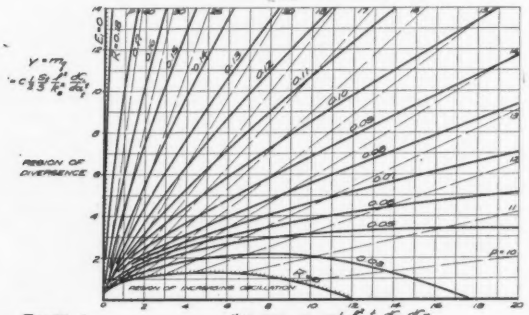

FIGURE 4

STABILITY DIAGRAM FOR GLIDING FLIGHT
CASE I $C_L = 1.0$
DAMPING FACTOR $= 1.11\sqrt{\frac{\rho}{\rho_0}}\,R$ PERIOD $= 0.43\sqrt{\frac{\rho}{\rho_0}}$ P SEC.

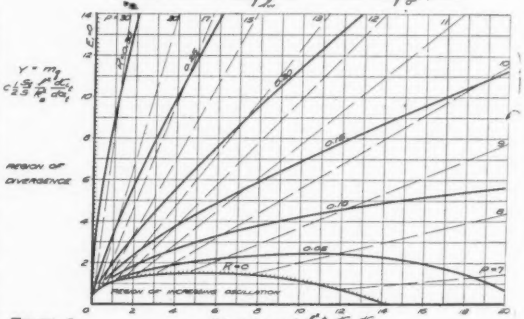

FIGURE 5

STABILITY DIAGRAM FOR GLIDING FLIGHT
CASE I $C_L = 1.4$
DAMPING FACTOR $= 0.94\sqrt{\frac{\rho}{\rho_0}}\,R$ PERIOD $= 0.53\sqrt{\frac{\rho}{\rho_0}}$ P SEC.

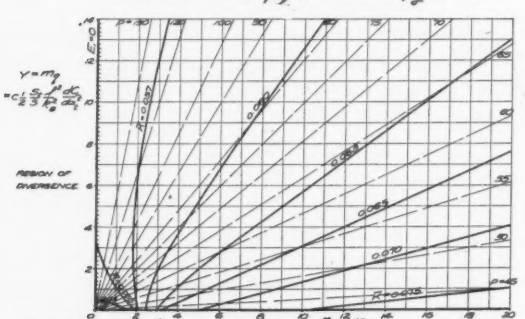

FIGURE 6

STABILITY DIAGRAM FOR GLIDING FLIGHT
CASE II $C_L = 0.2$
DAMPING FACTOR $= 2.48\sqrt{\frac{\rho}{\rho_0}}\,R$ PERIOD $= 0.20\sqrt{\frac{\rho}{\rho_0}}$ P SEC.

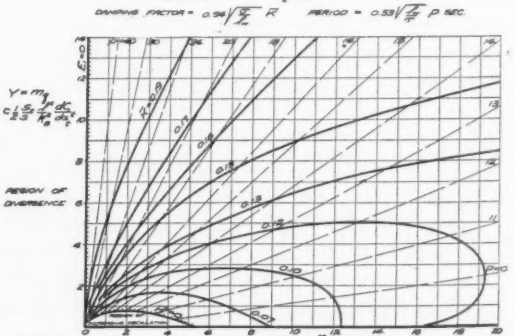

FIGURE 7

STABILITY DIAGRAM FOR GLIDING FLIGHT
CASE II $C_L = 1.0$
DAMPING FACTOR $= 1.11\sqrt{\frac{\rho}{\rho_0}}\,R$ PERIOD $= 0.43\sqrt{\frac{\rho}{\rho_0}}$ P SEC.

danger of instability due to increasing oscillation is more probable than at a low lift coefficient (corresponding to high speed). This indicates that dynamic instability is unlikely to occur for low lift coefficients. Closer study reveals that: (1) the damping factor decreases and the period increases with increase in altitude and wing loading, other quantities remaining constant; (2) the damping factor and period should be as large as possible in order to avoid the region of increasing oscillation; (3) a large k_B, corresponding to a large moment of inertia about the lateral axis of the airplane, increases the danger of instability thus leading to the conclusion that the distribution of weight along the longitudinal axis should be as compact as possible; (4) an increase in tail length, as well as an increase in S_t, increases the dynamic stability.

It is seen that if we assume Y remains the same, increasing the static stability (dC_M/dC_L increasing negatively) causes the period to decrease. In order to avoid simultaneously a period which is undesirably low and the two regions of instability, it is preferable to have a moderate degree of static longitudinal stability obtained with a large horizontal surface area, S_t, in order to make Y large. It is also evident that even though we have static stability (dC_M/dC_L negative) we may have dynamic instability if C_L is high and the combination of other variables is such that the value of Y is small. (See Figs. 4 and 5.)

To be reasonably certain of stability in power-on flight we should refer to a family of charts developed similarly to the present ones which would include the effects of changes in the various stability derivatives brought about by the thrust and the propeller slip-stream. Before such an additional group of curves can be successfully developed, however, much additional research on the effects of propeller slip-stream on tail surfaces and general aerodynamic characteristics is required.

Conclusions Important to Design

Important conclusions to keep in mind for design use are briefly summarized as:

(1) The airplane should have a moderate static stability with relatively large horizontal surfaces. (2) The moment of inertia about the lateral axis should be as small as possible. (3) For reasonable assurance that the plane will be stable in power-on flight, the boundaries of gliding stability should not be approached too closely.

Example of Use of Charts

As an illustration of the method for using the charts, consider the Douglas Transport DC–2, the properties of which are the following:

$W = 18{,}000$ lbs. $\qquad \dfrac{dC_L}{d\alpha} = 4.7,$

$S = 939$ sq. ft., $\qquad \dfrac{dC_M}{dC_L} = -.14,$

$S_t = 154.6,$ $\qquad l = 36.6$ ft.,

$l_w = 19.2$ lbs./sq. ft. $\qquad t = 11.86$ ft.,

$l/t = 3.09,$ $\qquad C_{D_P} = 0.02,$

$k_B{}^2 = 45.0,$ $\qquad \dfrac{dC_{L_t}}{d\alpha_t} = \dfrac{5.5}{1 + \dfrac{5.5}{\pi \text{ A.R.}_t}} = 4.0.$

π A.R.$_t = 14.45,$

Procedure

(1) Since C_{D_P} for the DC–2 is of the order of 0.02 and $dC_L/d\alpha$ is 4.7, the charts developed for Case I are used. The airplane is considered flying at a 5000 ft. density altitude.

(2) Coordinates are:

$$X = \mu m_w = -\left[13.08 \frac{l_w}{\sigma_{5000} l}\right]\left[\frac{1}{2}\frac{l^2}{k_B{}^2} - \frac{t}{l}\frac{dC_L}{d\alpha}\frac{dC_M}{dC_L}\right]$$

$$= -\left[\frac{13.08 \times 19.2}{.862 \times 36.6}\right]\left[\frac{1}{2}\frac{(36.6)^2}{45} - \frac{1}{3.09}(4.7)(.14)\right]$$

$$= -25.3;$$

Note: The minus sign has been omitted on figures.

$$Y = m_q = C\frac{1}{2}\frac{S_t}{S}\frac{l^2}{k_B{}^2}\frac{dC_{L_t}}{d\alpha_t}$$

$$= 1.2\left[\frac{1}{2}\frac{154.6}{939}\frac{(36.6)^2}{45}(4)\right] = 11.7.$$

(3) Referring to the charts for Case I (Figs. 2, 3, 4 and 5) and using the coordinates X and Y from above, we see that the airplane is well within the stable range. The values for damping factor and period read from the charts (extrapolating for X) are given in the following table and the final values are plotted in Fig. 8. The time to damp to one-half amplitude is also plotted.

Figure	C_L	p	P(sec.)	K	D.F.	$T_{\frac{1}{2}}$
2	0.2	64	60.5	.0275	.0145	47.8
3	0.6	21.5	35.6	.0440	.0137	50.6
4	1.0	13.0	27.7	.0800	.0188	36.9
5	1.4	9.3	23.3	.1400	.0279	34.8

$(\sigma/l_w)^{\frac{1}{2}}=0.212$; $(l_w/\sigma)^{\frac{1}{2}}=4.725$.

From flight tests the DC–2 proved dynamically stable with a period for the longitudinal oscillation of approximately 50 seconds, $C_L \doteq 0.3$. The period curve of Fig. 8 indicates a period at 5000 ft. density altitude of approximately 50 seconds, a value which agrees well with that measured.

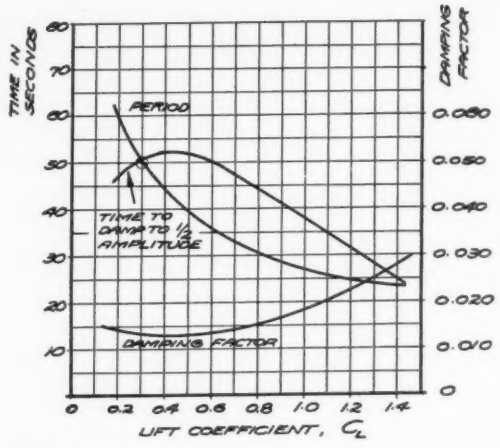

FIGURE 8

VALUES OF DAMPING FACTOR, PERIOD, AND TIME TO DAMP TO ONE HALF AMPLITUDE FOR THE DOUGLAS TRANSPORT DC-2, GLIDING FLIGHT.

Optimum Flight Path in Air Transport Operation

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WITH the introduction of the modern high performance airplane into the air transportation systems of the country, it has become increasingly necessary to make some study of the factors which influence the proper selection of flight paths for the efficient operation of the aircraft. In the past it has been the practice of the operators to make merely a rough estimate of the effect of the winds to determine the best altitude at which to fly on the course. There has been, in general, a complete neglect of the variations in the performance of the aircraft. It is the purpose of this paper to present, in so far as possible, a means which is practical from the standpoint of the operator for determining the most efficient flight path.

The problem, in general, is the following: To provide a means of quickly determining before the start of a flight the path which the pilot should follow in order to fly between any two points in the minimum possible time. Thus, for a given set of cruising specifications for the airplane, this path becomes the optimum not only with regard to elapsed time but also for economy of operation. First, it will be assumed that the flight will be restricted to the vertical plane, which includes the starting point and the destination. The flight path will then have two dimensions in which to vary, with an infinite number of possibilities from which to choose.

Let us assume that the complete performance characteristics of the airplane are known, either from flight test data or from performance estimates based on calculated values. If the plane were free to start the flight at any altitude directly above the starting station and to end the flight at any other altitude above the destination, then the correct path would be determined by the combination of winds and cruising air speed of the airplane in level flight which would give the maximum ground speed. The problem would be one of solving the wind triangle for various altitudes along the path and picking the maximum. The calculation of this effective ground speed has been simplified in such a manner that it can be presented in a single compact table from which the effective ground speeds can be computed quite rapidly. This is given in Table I and is based on the following equation:

$$V_e = V_c + \Delta V_e,$$

where

$$\Delta V_e = W \cos\theta - \tfrac{1}{2}\frac{(W \sin\theta)^2}{V_{c\ \text{ave.}}},$$

and where $V_{c\ \text{ave.}}$ is the average of the cruising air speeds of the particular plane in level flight for various altitudes. The remaining quantities are defined in Fig. 1. This equation has been derived by neglecting factors which introduce maximum error of less than 1 percent, and this is in keeping with the accuracy of the wind velocities as determined by the upper air balloon soundings. In Table I the values of ΔV_e are given as a function of wind direction, θ, and wind velocity, W; and the cruising air speeds are given as a function of altitude alone. In the actual calculation the values of this ground speed will be determined from the table and entered at the corresponding position on the flight path chart (Fig. 2). The path to be chosen will be that which passes through the maximum values of V_e.

In the practical case, such a flight path as presented above is not possible, since the plane must first climb to this altitude from the starting point and finally glide into the destination. It will be assumed that the flight is of sufficient length that it will be practical to climb to this

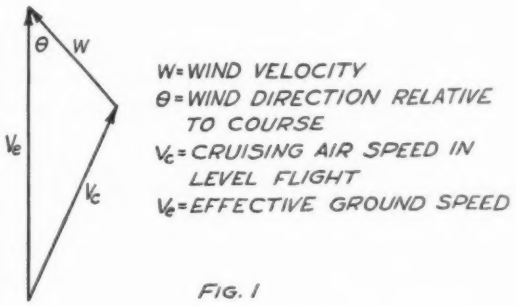

FIG. 1

TABLE I. *Effective cruising speed of the Douglas DC-2 (65% full power).*

Table of ΔV_c

W	$\theta=0$	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180
2	2	2	2	2	2	1	1	1	0	0	0	−1	−1	−1	−2	−2	−2	−2	−2
4	4	4	4	4	3	3	2	1	1	0	−1	−1	−2	−3	−3	−4	−4	−4	−4
6	6	6	6	5	5	4	3	2	1	0	−1	−2	−3	−4	−5	−5	−6	−6	−6
8	8	8	8	7	6	5	4	3	1	0	−2	−3	−4	−5	−6	−7	−8	−8	−8
10	10	10	9	9	8	6	5	3	2	0	−2	−4	−5	−7	−8	−9	−9	−10	−10
12	12	12	11	10	9	8	6	4	2	0	−2	−4	−6	−8	−9	−11	−11	−12	−12
14	14	14	13	12	11	9	7	4	2	−1	−3	−5	−7	−9	−11	−12	−13	−14	−14
16	16	16	15	14	12	10	8	5	2	−1	−3	−6	−9	−11	−13	−14	−15	−16	−16
18	18	18	17	15	14	11	8	6	2	−1	−4	−7	−10	−12	−14	−16	−17	−18	−18
20	20	20	19	17	15	12	9	6	3	−1	−5	−8	−11	−14	−16	−18	−19	−20	−20
22	22	22	21	19	16	14	10	6	3	−1	−5	−9	−12	−15	−17	−19	−21	−22	−22
24	24	24	22	20	18	15	11	7	3	−1	−6	−10	−13	−16	−19	−21	−23	−24	−24
26	26	26	24	22	19	16	12	7	3	−2	−6	−10	−14	−18	−21	−23	−25	−26	−26
28	28	28	26	24	21	17	13	8	3	−2	−7	−11	−16	−19	−22	−25	−27	−28	−28
30	30	30	28	25	22	18	13	8	3	−2	−7	−12	−17	−21	−24	−27	−29	−30	−30
32	32	31	30	27	23	19	14	9	3	−3	−8	−13	−18	−22	−26	−28	−30	−31	−32
34	34	33	32	29	25	20	15	9	3	−3	−9	−14	−19	−24	−27	−30	−32	−33	−34
36	36	35	33	30	26	21	16	9	3	−3	−9	−15	−20	−26	−29	−32	−34	−35	−36
38	38	37	35	32	28	22	16	10	3	−4	−10	−16	−22	−27	−31	−34	−36	−37	−38
40	40	39	37	34	29	23	17	10	3	−4	−11	−17	−23	−28	−32	−36	−38	−39	−40
42	42	41	39	35	30	24	18	11	3	−4	−12	−18	−24	−30	−34	−38	−40	−41	−42
44	44	43	41	37	31	26	18	11	3	−5	−12	−19	−26	−31	−36	−39	−42	−43	−44
46	46	45	43	38	33	27	19	11	3	−5	−13	−20	−27	−33	−37	−41	−44	−45	−46
48	48	47	44	40	34	28	20	11	3	−6	−14	−22	−28	−34	−39	−43	−46	−47	−48
50	50	49	46	42	36	28	20	12	3	−6	−15	−23	−30	−36	−41	−45	−48	−49	−50
52	52	51	48	43	37	29	21	12	2	−7	−16	−24	−31	−37	−43	−47	−50	−51	−52
54	54	53	50	45	39	30	22	12	2	−7	−17	−25	−33	−39	−45	−49	−52	−53	−54
56	56	55	52	47	40	31	22	12	2	−8	−17	−26	−34	−41	−46	−51	−54	−55	−56
58	58	57	54	48	41	32	23	12	2	−8	−18	−27	−35	−43	−48	−52	−56	−57	−58
60	60	59	55	50	42	33	23	13	2	−9	−19	−28	−37	−44	−50	−54	−57	−59	−60

Cruising Air Speed V_c

H(ft.)	V_c	H(ft.)	V_c
0	−166	9000	−181
1000	−170	10000	−183
2000	−171	11000	−184
3000	−173	12000	−185
4000	−174	13000	−187
5000	−176	14000	−188
6000	−177	15000	−190
7000	−178	16000	−191
8000	−180	17000	−193

previously determined altitude. Knowing the performance of the airplane in question, it is easily possible to construct curves of altitude against distance for various constant rates of climb from the starting point and similarly curves for constant rates of glide into the destination. It is necessary to know only the cruising air speed as a function of the altitude and the rate of climb or glide. In determining these curves, we neglect the effect of the wind for which we will correct later. These curves are plotted on the flight path chart (Fig. 2). Let us first consider the climb curves. Having determined these curves for various constant rates of climb, the time required to fly from the starting point of the flight to any point on any one of the curves can be calculated and is equal to the difference in altitude divided by the rate of climb, since the rate of climb for each curve was kept constant. The time required to fly in *level flight* from a point directly above the starting station to the point on the climb curve can also be calculated. The difference between these values will then be the time lost by reason of having to climb. These values of time lost are calculated for each 1000 feet on each of the curves and the values recorded alongside the curves on the flight path chart (Fig. 2). If the same calculation is made for the glide curves, the result will be similar except that the calculated time differences will be time gained during the glide. These curves need be made up only once for a given type of plane and a given route, and are printed on the basic chart to be used for each individual flight.

This calculation was made for the Douglas DC-2 Transport operating at 65 percent power, and the curves, together with the time lost or gained, were superimposed on the flight path chart (Fig. 2). It will be noticed on this chart,

however, that no rate of climb curves appear. This was by reason of the fact that for this particular airplane the time lost in climb was practically independent of the rate of climb and the values were themselves small. In the table the recorded time differences are in minutes.

The above calculations now give, if the effect of the winds during the climb and glide is neglected, a practical means of determining the optimum flight path for a given set of meteorological conditions. It is necessary merely to determine first the altitude at which to fly between the climb and glide and then to select the rate of climb which will give the minimum time lost in the case of the climb, and the maximum time gained in the case of the glide. These values are given alongside the climb and glide curves. However, the variance in the winds aloft may alter this selection, and we must make a correction accordingly. Assume that the vertical distribution of the effect of the winds on ground speed is linear with altitude, in other words that ΔV_e is a linear function of the altitude. This assumption is justified by reason of the fact that the corrections due to the winds are comparatively small so that variations from such a distribution will be negligible. The only condition of wind distribution which can affect time lost or gained during the climb or glide will be that in which the wind velocity at the ground differs from that aloft. In this case, the correction will be directly proportional, with the assumption of linearity, to the difference in the velocities at or near the ground and aloft. This correction in minutes per 1000 feet of climb is given by the equation.

$$\frac{\Delta t}{\Delta H} = \frac{\delta(\Delta V_e)}{CV_{e\ \text{ave.}}} \times 500,$$

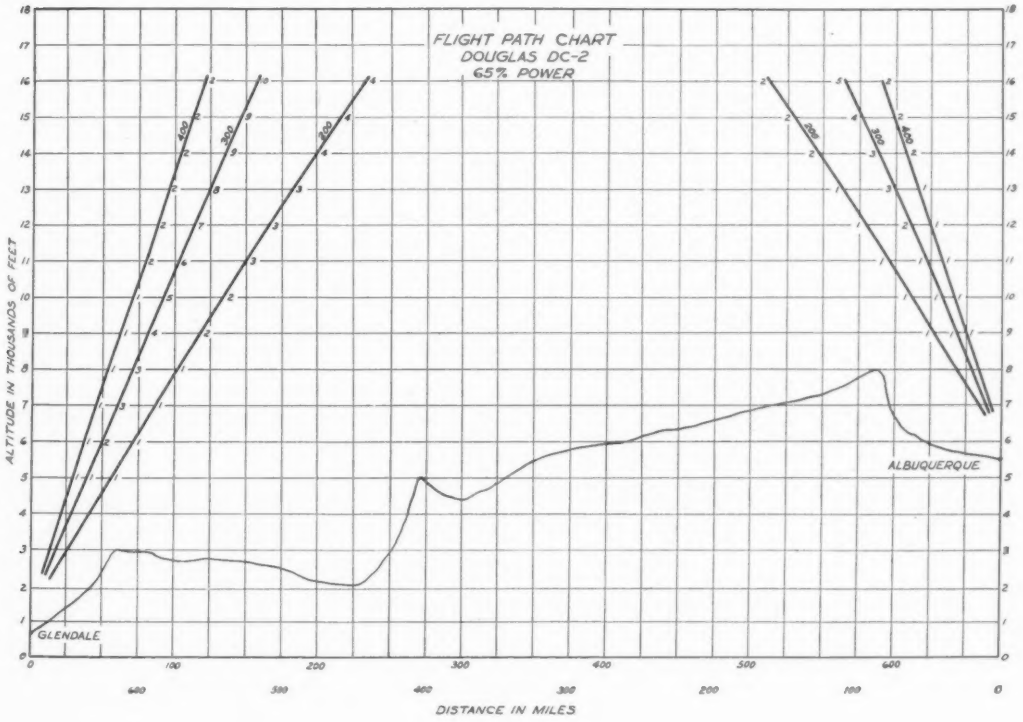

FIG. 2.

where

C = rate of climb in feet per minute,
$V_{e\ ave.}$ = average cruising air speed (in m.p.h.) of the particular plane in question,
$\delta(\Delta V_e)$ = value of ΔV_e at the altitude of the intermediate path minus the value of ΔV_e at the ground,
ΔH = altitude (in thousands of feet) through which the plane must climb or glide.

This equation has been made up into Table II and the values given in this table are the corrections in time lost or gained in minutes per 1000 feet of climb or glide. In using this table the operator will, after determining the correction for each of the climb and glide curves at the altitude of the intermediate flight path, alter the selection of the optimum rate of climb or glide correspondingly. If ΔV_e has a higher value aloft than at the ground, the correction will be additional time lost, and if less, time gained.

TABLE II. *Time correction in climb and glide (minutes per 1000 ft. of climb).*

$\delta(\Delta V_e)$	Rate of Climb (ft./min.) = 100	200	300	400
5	0.1	0.1	0.0	0.0
10	0.3	0.1	0.1	0.1
15	0.4	0.2	0.1	0.1
20	0.5	0.3	0.2	0.1
25	0.7	0.4	0.2	0.2
30	0.8	0.4	0.3	0.2
35	0.9	0.5	0.3	0.2
40	1.0	0.5	0.4	0.3
45	1.2	0.6	0.4	0.3
50	1.3	0.7	0.4	0.3

Finally, we must correct the position at which the pilot must start his glide in order to come into the destination at the proper altitude. This correction is similar to the time correction and is given by the equation

$$\frac{\Delta s}{\Delta H} = 16.67 \frac{\Delta V_{e\ ave.}}{C},$$

in which $\Delta s/\Delta H$ is the value of the correction in miles per 1000 feet of glide, and $\Delta V_{e\ ave.}$ is the average of the values of ΔV_e at the ground and that at the altitude of the intermediate flight path. ΔH is the same as before. The values of this correction are given in Table III. In using this correction, if the value of $\Delta V_{e\ ave.}$ is positive (i.e., a tail wind), the pilot must start the glide a distance Δs earlier than that given on the flight path chart.

TABLE III. *Climb and glide position correction (miles per 1000 ft. of climb).*

ΔV_e Average	Rate of Climb (ft./min.) = 100	200	300	400
5	0.8	0.4	0.3	0.2
10	1.7	0.8	0.6	0.4
15	2.5	1.3	0.8	0.6
20	3.3	1.7	1.1	0.8
25	4.2	2.1	1.4	1.1
30	5.0	2.5	1.7	1.3
35	5.8	2.9	1.9	1.5
40	6.7	3.3	2.2	1.7
45	7.5	3.7	2.5	1.9
50	8.3	4.1	2.8	1.2

In summary the procedure which will be followed by the operator will be the following:

(1) Using the upper air balloon soundings, determine the values of the effective ground speed from Table I, and record these at their corresponidng positions on the flight path chart. In addition, it will be necessary to record the values of ΔV_e for the balloon soundings at the start and destination of the flight.

(2) Draw a curve through the points of maximum effective ground speed as determined in (1). It is assumed that the climb or glide taking place during this intermediate flight path will have a negligible effect on the effective ground speed. This assumption is well within the limits of the accuracy of the values as determined, since the rates of climb and glide seldom exceed 50 feet per minute.

(3) Correct the time lost or gained by Table II, for each of the climb and glide curves at their intersection with the intermediate flight path curve. From this, determine the optimum rates of climb and glide.

(4) Correct, by means of Table III, the position at which to start the optimum rate of glide determined in (3).

The results made available by the use of the above analysis present another interesting possibility. This is the problem of scheduling aircraft operations. In other words, given a particular airplane to be flown over a given route with certain existing conditions of wind, how should

the power be regulated in order to make the flight in the scheduled time? For conditions in which there are excessive tail winds, it would be desirable to reduce the power in such a manner that the flight might be made in the scheduled time. In the case of head winds, the problem would be one of increasing power by a corresponding amount. If it is assumed that the power required is directly proportional to the cube of the velocity, we obtain the following equation:

$$\frac{P}{P_0}=K\left[\frac{S}{T-\Delta t-t}-\frac{\Delta V_e}{60}\right]^3,$$

in which

P/P_0 = ratio of power required to full power,
S = length of flight in miles,
T = scheduled time for the flight in minutes,
Δt = time lost in climb and glide plus time lost due to late starting,
t = time lost in take-off and landing (taken as an average value),
ΔV_e = the average value for the intermediate flight path as determined in the previous analysis.

K is a constant determined for the particular airplane in question and is based upon the approximate formula $P/P_0=KV_c^3$. This equation has been incorporated into a chart (Fig. 3). In using the chart, enter it with the value of T' (defined on the chart), going vertically to the length of the flight S, and then horizontally to the value of ΔV_e (average value for intermediate path) for the particular flight in question. This gives the percentage power required to complete the flight in the scheduled time. This particular chart, together with the previous charts and tables, was made up for the Douglas DC–2 Transport. The heavy dashed lines indicate the limits of power available at the altitudes specified due to limitations of manifold pressure and r.p.m. imposed by the motor manufacturers.

During the past year the author, through the cooperation of Transcontinental & Western Air, Inc., was enabled to try out the method as applied to airline operation. Two routes were covered, one from Pittsburgh to Newark, a distance of 300 miles, and the second from Glendale to Albuquerque, a distance of 700 miles. The first route was, of course, much too short to obtain full advantage of such a system as this, since in most cases the flight path consisted of a

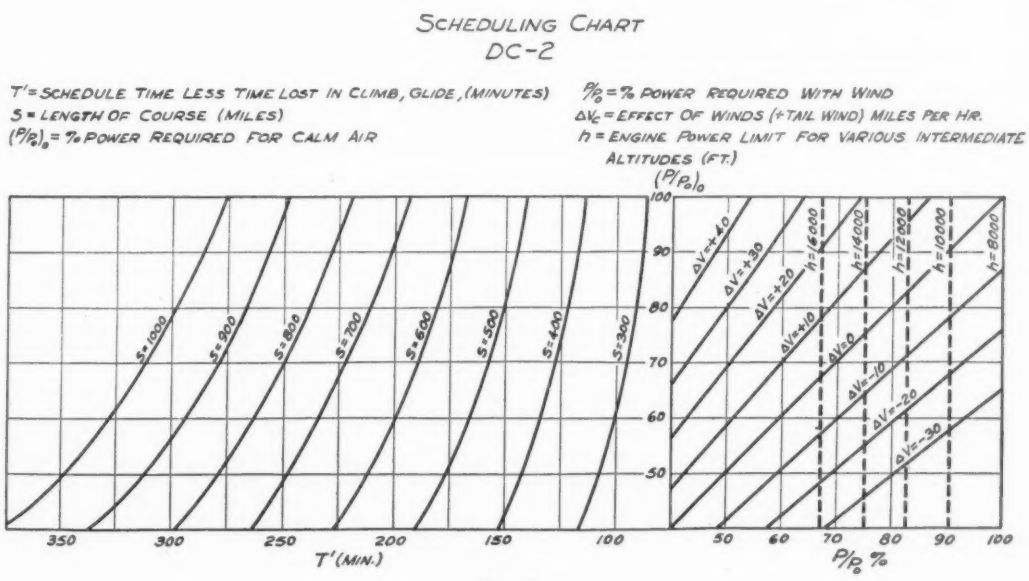

FIG. 3

climb followed immediately by a glide, with no intermediate flight path in between. However, one important observation was made. The pilots were enabled, by the use of the charts, to accurately determine the point at which to start their glide such that they arrived at the destination quite accurately at the required 500 foot level.

The second run from Glendale to Albuquerque was, of course, somewhat longer and advantage could be taken of the intermediate flight path. It was found that a maximum of ten minutes was required to make up the flight path chart for a given trip. Thus the time required for such an operation is not prohibitive from the practical standpoint. It is, of course, rather difficult to determine just what advantage is gained by the use of such a system, since there are no data with which to compare it. However, in every case, the flights were made in scheduled time or better, so that some advantage must have been gained by the use of the method.

In conclusion, the author wishes to point out several limitations which are imposed at present due to lack of sufficient data. First, due to lack of wind data above the lower level of the clouds, the use of the system is restricted to periods in which the intermediate flight path occurs below the ceiling. Second, in most cases there is a lack of sufficiently recent balloon sounding data to assure accuracy of the flight path as determined. However, with the extension of data to higher levels and to cloudy conditions, it is expected that such a method as this will have an important application, particularly when higher altitudes are attained in air transport work.

The Scale of the Boundary Layer in the Atmosphere

ROBERT H. GODDARD AND ARNOLD M. KUETHE, *Daniel Guggenheim Airship Institute*

(Received September 22, 1934)

THE flow in the boundary layer along the earth's surface has been the subject of many investigations. The extent of its similarity with turbulent boundary layers in channels and along flat plates has yet to be established, however, primarily because of the disturbing influences of the temperature lapse rate and the Coriolis force.

The aim of the present paper is to give some evidence pointing to a method of approach to the problem of atmospheric turbulence along a line suggested by recent advances in the study of small scale boundary layers.

Records of pilot balloon ascensions made by the staff at the U. S. Weather Bureau station at Akron, Ohio, show, in about fifty percent of the cases, a well-defined boundary layer varying in thickness from 200 to, in a few cases, over 1000 meters. The author is indebted to Mr. L. J. Fangman, meteorologist at that station, for the records necessary to carry out this investigation. An analysis of these records has been made, considering those quantities which determine the flow in turbulent boundary layers.

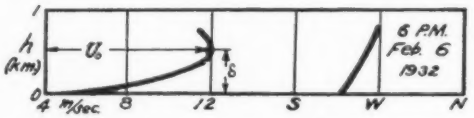

FIG. 1. Record of balloon ascension showing velocity and direction of the wind as a function of the height above the ground.

For fully developed turbulent flow, two quantities, δ, the thickness of the boundary layer, and U_0, the velocity at the outer edge, determine the mean profile. These quantities then determine the scale of the mean flow. If we consider U_0/δ, we have a quantity with the dimension

one/time,

the same as that of frequency.

Fig. 1 is a typical balloon ascension record, in which both velocity and direction are shown as functions of the height. Due to the earth's rotation the true frictional layer extends slightly beyond the point of maximum velocity. In this preliminary investigation however it was considered justifiable to take δ as shown in Fig. 1. The quantity U_0/δ was computed for 270 of these runs and the results, plotted in Fig. 2 in percentage of the total number of observations contained in intervals of 0.003 in U_0/δ, show a definite grouping of the values about $U_0/\delta = 0.026$. The balloon ascensions were observed morning and evening (6 A.M. and 6 P.M.). There was however, no noticeable shifting of the maximum of the percentage frequency curves for the two groups of observations. In the cases considered, the wind direction at the point where U_0 is measured also had a negligible effect.

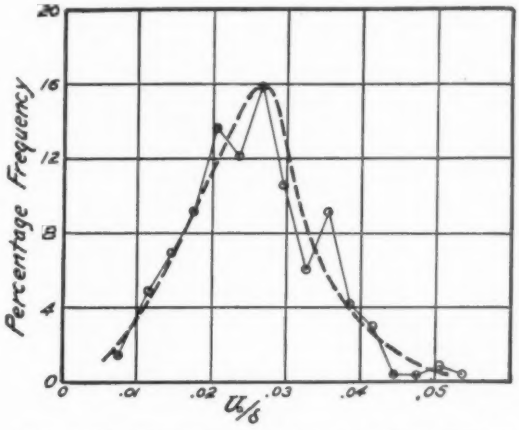

FIG. 2. Percentage frequency (percent of total number of observations contained in intervals of 0.003 in U_0/δ) curve for U_0/δ (1/seconds).

Dividing U_0/δ by a frequency results in a dimensionless quantity, $U_0/n\delta = B$. The frequency of velocity fluctuations in the atmosphere, as in turbulent flow in channels, is of a statistical nature, and, consequently, n must be an *average* frequency over a time interval which is long compared with the average period of the fluctuations. Since U_0/δ itself shows a definite grouping, a band of frequencies, independent of

this quantity, might still show a grouping of B. Therefore, in order for B to have physical significance, there should exist a band of frequencies for which a percentage frequency curve for B shows a tighter grouping than for all other frequency bands.

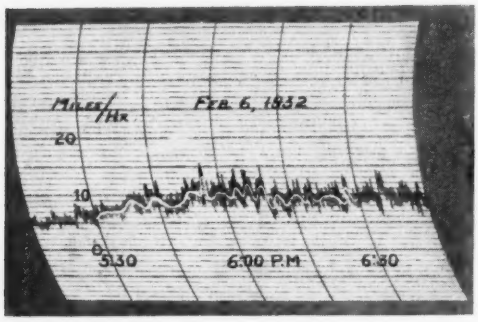

FIG. 3. Photograph of anemometer record. The white line through the center of the band indicates the frequency band considered.

Records from a continuously recording anemometer at the Akron Airport (the anemometer cups are 50.7 feet above the ground), have been examined with a view to finding such a frequency band. Fig. 3 is a section of a record, and the white line drawn through the center of the band of faster fluctuations defines a frequency band in which periods vary between approximately 2 and 8 minutes. The number of these fluctuations per hour (one-half hour on either side of the corresponding balloon ascension) was counted for 170 cases. The result is shown in percentage frequency in Fig. 4. The grouping of B for these 170 cases is shown in Fig. 5.

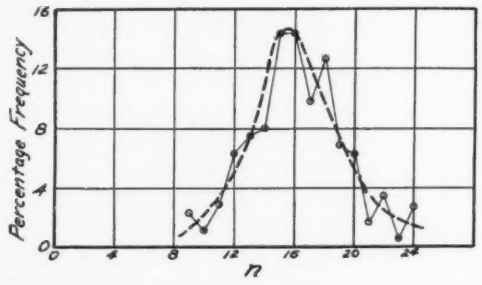

FIG. 4. Percentage frequency (percent of total number of frequency counts for each whole number per hour) curve of n (1/hours).

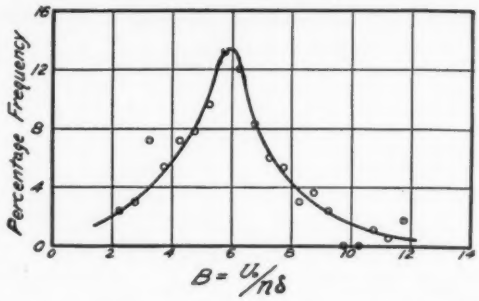

FIG. 5. Percentage frequency (percent of total number of observations contained in intervals of 0.5 in $U_0/n\delta$) curve of $U_0/n\delta$.

In order to be able to fix on the frequency band giving the *tightest* grouping of B, it will be necessary to take balloon ascension records for different speeds of the wind speed recorder. In this way, over a given length of record, different frequency bands will be accentuated and it may be possible, with a large number of observations, to separate that band characteristic of U_0/δ.

It is interesting to observe that in turbulent flow in channels, frequencies exist for which B has a value of the same order as that about which the values in Fig. 5 group. Oscillograms of the velocity fluctuations taken during the course of an investigation by Wattendorf and the author at the California Institute of Technology[1] indicate that frequencies of the order of 100 per second make an important contribution to the root mean square value of the fluctuations. In this case $U_0=14$ meters per second, and $\delta=0.025$ meter. The corresponding value of B is 5.6. Reichardt,[2] in carrying out a similar investigation, observed that frequencies of the order of 1.5 per second contributed greatly to the r.m.s. value of the velocity fluctuations. For this case, $U_0=1.1$ meters per second and $\delta=0.125$ meter, giving B a value of 5.9.

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[1] F. L. Wattendorf and A. M. Kuethe, *Investigation on Turbulent Flow by Means of the Hot Wire Anemometer*, Physics **5**, 6, 153–164 (1934). (Figure 11 is an oscillogram of the velocity fluctuations.)

[2] H. Reichardt, *Die quadratischen Mittelwerte der Laengsschwankungen in der turbulenten Kanalstroemung.* Zeit. f. Angew. Math. u. Mech. **13**, 3, 177–180 (1933).

Since U_0/n is the wave length of the disturbance at the edge of the boundary layer, there apparently exists an important band of frequencies with wave lengths at the edge of the boundary layer of the order of six times the boundary layer thickness.

It is expected that an approach to the problem of atmospheric turbulence from the standpoint of the boundary layer, may lead to results independent of the surrounding terrain, except insofar as it affects the mean velocity distribution.

Records of the three components of the velocity fluctuations, taken by means of a hot-wire arrangement on a blimp, at various altitudes, are now being made. The evaluation of these records will give further data on the extent of the similarity between atmospheric and small-scale boundary layers.

RESUMÉ

The scale of the atmospheric boundary layer is investigated statistically. It is found that values of U_0/δ, where U_0 is the velocity at the outer edge of the boundary layer and δ is the boundary layer thickness, cluster about a definite value.

A percentage frequency curve of $B = U_0/n\delta$, where n is the frequency of a certain band of velocity fluctuations, using a frequency band which stands out on the anemometer records analyzed, is given, and there is again a grouping of the values.

The values group about $B = 6$ (approximately), and values of B calculated from recent researches in turbulent flow in channels are of the same order.

Book Review

Our Wings Grow Faster, by Grover Loening; Doubleday, Doran & Co., New York, 1935; 203 pages; illustrated; $3.75.

Those who wish to learn more about the beginnings of aeronautical engineering in the United States will be greatly interested in this book, which is written by one of the earliest participants in the aeronautical development of this country. As the first student to receive a degree for a thesis on aeronautics as well as the first aeronautical engineer to be employed by the Army, Mr. Loening writes with first-hand knowledge of the events which laid the foundations of aviation in America.

The book is frankly autobiographical and opinions on events, persons, and products are expressed with a directness which has always been characteristic of pioneers in any endeavor. Everyone who reads the ninety-two chapters will sense the gleeful spirit with which the author writes praise and criticism alike.

And he is not sparing in the latter. Running through the book is a thread of argument for the aircraft constructors *per se* as opposed to manufacturers of other products who have attempted to build aircraft. It is apparent that Mr. Loening holds a strong resentment against the treatment that he, Martin, Vought, and others received during the War.

The illustrations, which are numerous, were selected with discrimination and add greatly to the interest of the text. All, apparently, were chosen to give those who entered aviation within the last few years a series of snapshots of persons and events which are now part of the early aviation history. The story of the early activities of the company formed by the Wright Brothers, written by one who participated in the work, is related with an intimate and friendly knowledge which is almost romantic in its interest. There are few of the original Wright group now in aviation and none who has the facility of expression of Mr. Loening.

The section on the San Diego period of the early Army fliers, using the primitive equipment then available, gives a record of the work of these early pilots after whom many of the Army air fields are named, as well as of those whose names have become famous in American aviation history.

From these beginnings, the book follows the development of aeronautics in this country up to the present day. A forceful opinion is expressed on practically all of the controversial questions which have agitated the aeronautical group. The struggles of the aircraft constructors before and during the War, Congressional investigations, famous aeronautical episodes, and a witty account of the 1929 aircraft stock market boom all pass in review.

"Our Wings Grow Faster" will undoubtedly become a classic of aviation history. Few will read the book without challenging many of its statements. Pioneers in any field have usually experienced grief and hardship. It is pleasant to read of one whose successful contributions to the art and science of aeronautics have brought him not only honors and distinction, but also the more substantial rewards.

His "War Baby" book that made him nearly $100,000 profit is one of the most remarkable instances of unexpected rewards in the history of publishing. His many friends among our members, knowing of his generous donations to the Institute, will wish for his new literary effort a similar success. He is one of the members of the Institute's Endowment Committee!

LESTER D. GARDNER

Periodically Interrupted Flow Through Air Passages

ALBERT C. ERICKSON, *Clark University*

(Received November 2, 1934)

THE purpose of this work was to study the pressure variations within channels moving rapidly through the air while the exit ends of the channels were being periodically opened and closed. The research had its inception in connection with questions raised as to what happens when a chamber (a rocket device), into which air may be admitted at the forward end, and from which it may be expelled from the rearward end by explosion, after being mixed with fuel, moves rapidly through the air. Such a device has been patented by Dr. R. H. Goddard, Head of the Physics Department at Clark University, U. S. Patent No. 1980266.

Since this patent application was filed, there have appeared two papers by G. A. Crocco[1] dealing with the problem of injecting fuel into air that has been compressed adiabatically by the passage of a chamber at high speed through the air. Crocco considered the chambers to move at constant velocity, however, and the pressure obtained at any particular speed would be much less than the maximum pressure produced by varying flow, at the same speed, as is demonstrated by the present investigation.

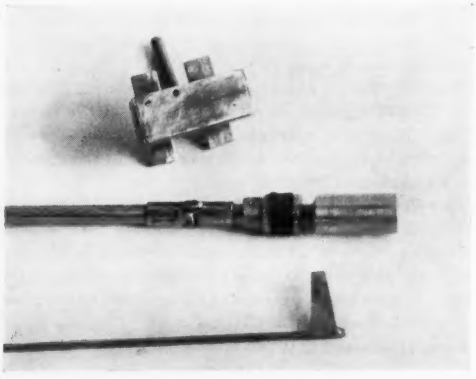


FIG. 1.

At excessively high speeds rocket propulsion is theoretically more efficient than engine and propellor propulsion; a fact which should make this method useful for high-speed travel in the stratosphere. Although the actual pressures measured in this work are small, the rate of increase of pressure with air speed is high, even for the relatively low air speeds used in this investigation. At the high speeds possible in stratosphere flight, very high pressures should be obtained. Reference may here be made to the great air pressures, due to surging, that occur in turbine-type air compressors; these pressures being, under proper conditions, sufficiently great to break off steel blades and throw them out against the stream of entering air.[2] One of the most important features of this method is that fuel alone is carried, and not a large extra weight of liquid oxygen, as is necessary for a rocket traveling at such a great height in the atmosphere that there is relatively little oxygen present.

The experimental work consisted of measuring at different instants of time the pressures existing at several positions within the air channels in which the flow of air was periodically interrupted by a rotating slotted disc.

A mechanical valve (see Fig. 1) consisting of a steel casing, closed at one end, in which a hollow brass cylinder was made to rotate in synchronism with the slotted disc by means of a simple gearing system, was used in obtaining the pressure magnitudes. Once during each rotation of the valve a narrow slot, 0.008 inch wide, in the brass cylinder was in exact coincidence with a similar slot in the steel casing. Communication with the manometer system was maintained by means of a 1/8 inch hole in the side of the brass cylinder moving in a groove on the inside surface of the steel casing, this groove affording at all times a path for the

[1] G. A. Crocco: Accad. Lincei Atti, 13, pp. 906–911, June 21, 1931; 14, pp. 161–166, Sept. 9, 1931.

[2] A. Stodola: *Steam and Gas Turbines*, translation by Loewenstein, Vol. 2, p. 1265, McGraw-Hill, 1927.

flow of air through a hole in the steel casing to the manometer. Figs. 2 and 3 show top and bottom views of tl base-board with the valve and steel spacing pieces in position.

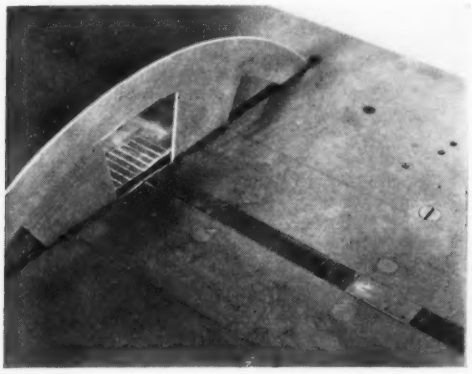


FIG. 2.

Pressure measurements were made at eight different instants of time within one complete pressure cycle. Considering 360° as a complete pressure cycle, the pressure measurements were taken every 45°; *viz.*, at 0°, 45°,—315°; these angular designations are subsequently referred to as the phase relations between the disc and the valve.

Due to the appreciable width of the slots in the steel casing and in the brass cylinder, the pressure measurements obtained were not actually instantaneous but were time averages over 18° of a pressure cycle. Hence a pressure magnitude listed at a 90° phase may be interpreted as a time average of the pressures existing during the interval from the position in the pressure cycle denoted by 81° to that denoted by 99°. Six hundred cycles per second was the maximum frequency of interruption at which pressure measurements were made in this research. At this frequency the valve furnished pressures which were time averages over only 0.00008 second.

The main body of the experimental data consists of pressure measurements taken at 45° phase intervals at five different positions within the channels used, and at frequencies of 50, 100, 150 and 250 interruptions per second and at air speeds of 25, 40, 50 and 55 miles per hour. The positions within the channels at which the measurements were made are designated by the numbers 1 to 5, 1 being the position nearest the entrance of the channels and 5 being the position closest to the exit end.

Because of lack of space, this paper presents only the briefest possible outline of method and results. For details, the reader is referred to the writer's thesis.[3] Five different channels were

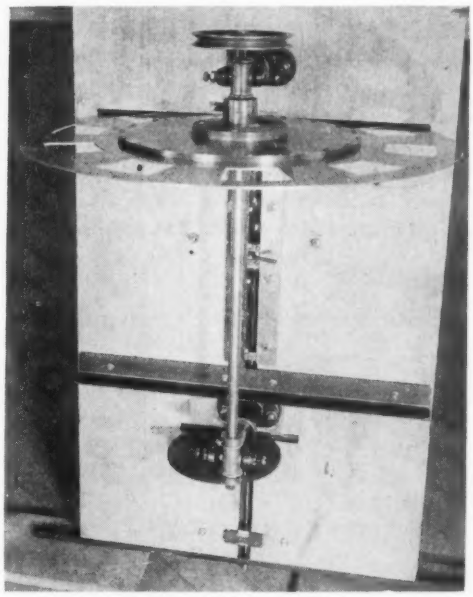


FIG. 3.

studied; this report deals only with one of them, a rectangular chamber 23 inches long, 1 and 27/32 inches wide and 2 inches high. All pressure magnitudes are expressed in terms of centimeters of water pressure; a negative sign preceding a pressure magnitude indicating that the pressure was below atmospheric pressure.

Fig. 4 shows the pressures obtained at a frequency of 250 interruptions per second and is a plot of 8 points on a pressure cycle lasting 1/250 second, each point representing a pressure which

[3] A. C. Erickson: *A Study of Periodically Interrupted Flow Through Air Passages*, Ph.D. Dissertation, Clark University, 1934.

occurred 0.0005 second later than that represented by the preceding point. At 55 miles per hour, then, the pressure increased by 31.5 cm during an interval of 0.0025 second and decreased by this amount in another 0.0015 second. This data was obtained at a point approximately midway in the channel.

Attention is called to the fact that because of the presence of the base-board, the pressures obtained were appreciably less than those which would actually occur if a complete channel, rather than a half-section, were used without a base-board. Also, it should be noticed that, since the pressure was not measured continuously along the cycle but at arbitrary phase positions, the actual maximum values may be of greater magnitude and may occur at other phases than those at which they are recorded.

Table I shows the variation of pressure changes

TABLE I.

Air speed M.P.H.	Frequency cycles/sec.	Ratios of Pressure Variation				
		No. 1	No. 2	No. 3	No. 4	No. 5
25	50.	2.47	2.65	2.94	3.65	5.24
25	75.	2.7	2.94	4.1	4.7	4.95
25	100.	3.47	3.58	2.71	5.5	4.1
25	150.	3.53	2.82	3.64	4.17	4.65
25	250.	6.46		6.55		7.15
40	50.	5.7	6.25	7.24	8.87	12.05
40	75.	7.25	7.95	11.1	12.25	13.3
40	100.	8.75	8.7	6.84	13.3	10.55
40	150.	8.7	7.4	8.4	10.9	11.2
50	50.	8.65	9.35	10.4	12.9	17.
50	75.	10.5	11.95	16.7	18.2	19.9
50	100.	12.8	13.2	9.8	18.65	15.55
50	150.	13.6	12.35	13.6	16.35	17.
50	250.	28.6		26.		22.4
55	250.	37.		32.9		25.8

within a cycle with change in position, frequency of interruption and air speed. It has been compiled from all the data for this channel of the type represented by Fig. 4. The unit of pressure variation used is the difference between the static pressures obtained at position 1 at an air speed of 25 miles per hour when the channel was fully open and when it was fully closed. The ratio of pressure variation is given in terms of this unit and is the maximum difference in pressure within the cycle divided by the unit of pressure variation. To illustrate, from Fig. 4 the static pressure

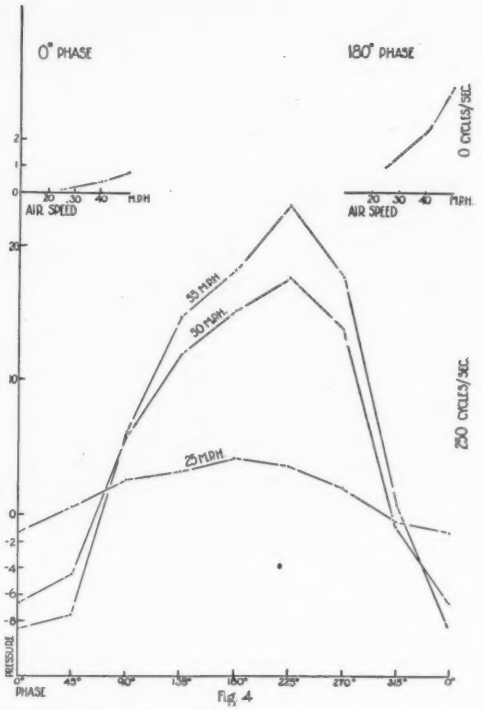


FIG. 4.

at position 1 at an air speed of 25 miles per hour and at a 0° phase was 0.1 cm and at a 180° phase, was 0.95 cm thus giving 0.85 cm as the unit of pressure variation. At an air speed of 55 miles per hour and a frequency of 250 interruptions per second, the maximum difference in pressure within the cycle was 31.5 cm resulting in a ratio of 37 (see Table I). This table definitely shows that the air speed was the predominating factor in increasing the pressure variation within any given pressure cycle.

Fig. 5 shows the variation of pressure with air speed for two different frequencies of interruption and also for the static case; the phase being 180° in all cases. It will be noticed that the pressure rises much more rapidly with increase in air speed during interrupted flow than during steady flow into the closed channel. Probably these curves are exponential curves and are actually steeper at high air speeds than is indicated for the reason that friction undoubtedly decreased considerably the air speed in the channels used, par-

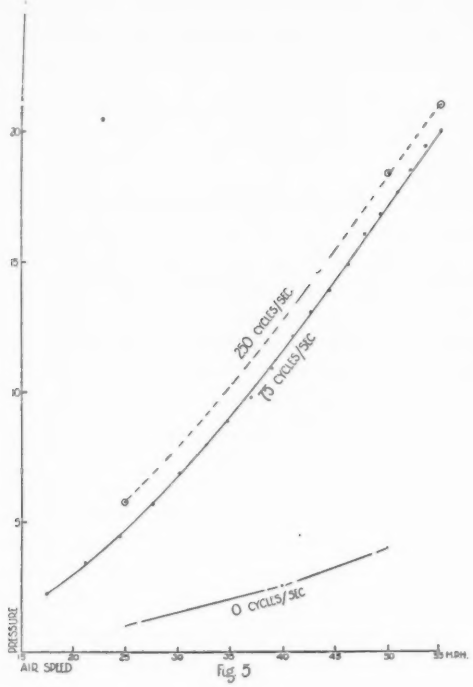


FIG. 5.

ticularly at high velocities.

At certain frequencies of interruption, differing with the channel used, the channels emitted definite characteristic tones usually of the same frequency as the frequency of interruption. This occurred at frequencies at which surging effects were present as manifested by unusually high pressure variations within a cycle. In the case of the long rectangular chamber here discussed, these sound effects were very marked at 75 and 250 cycles per second. Careful measurements showed definite maximum pressure values in the immediate vicinity of these frequencies and indicate the existence of an optimum frequency of interruption for a given channel.

The long rectangular channel, considered as a tube closed at one end, had a fundamental acoustic frequency of 142 vibrations per second and, considered as an open tube, a fundamental frequency of 284 per sec. (Actually, very marked resonance occurred with this channel at 75 and 250 interruptions per second.) The presence of the rotating slotted disc at the rear of the channel obviously precludes consideration of the channel as either a stopped or an open tube. It is possible that the end effects introduced by the rotating slotted disc might cause the channel to respond to oscillations other than its acoustic fundamentals.

The end effects added to the channel by the rotating slotted disc might be considered as adding acoustic impedance to the system, thus causing a decrease in the fundamental resonance frequency. This suggestion appears plausible in view of the fact that, in acoustics, the acoustic impedance at the closed end of a stopped tube is infinite. Attention is called to the fact that in this work the rear of the channels was periodically fully closed or stopped and hence the acoustic impedance at the rear of the chamber actually did become practically infinite periodically. The question arises whether or not the periodicity of the impedance could be expressed as a sine function of the time (sin $1/2\omega t$) so far as inertia, stiffness and resistance are concerned, and also whether or not the methods of partial differential equations, using normal functions which would not necessarily have to be orthogonal, for free vibrations with periodic resistance increasing to infinite values periodically, could be used to advantage.

Gibson[4] discusses the action of a water hammer and shows that if closure of a pipe line occurs in a time less than $1/V$ seconds, where V is the velocity of propagation of sound in the medium, the pressure rise will be the same as if closure were instantaneous.

At a frequency of 600 interruptions per second, the disc rotated from a position completely opening the channel to a position completely closing it in 1/1200 sec., thus satisfying the above condition.

The measured pressure, however, differed greatly from that calculated by Gibson's formula. This was to be expected since the periodically interrupted flow through the channel differed

[4] C. V. Drysdale and others, *The Mechanical Properties of Fluids*, Blackie & Son, Limited, London, 1925.

quite radically from a steady flow which was suddenly stopped, there being no doubt but that the periodic interruption of the air stream reduced appreciably the velocity with which the air flowed through the channel during the short intervals of time at which the channel was unobstructed.

If, as in the case of the hydraulic ram, the pressure rise at the rear of the channel, when it is suddenly closed, is propagated with the velocity of sound toward the entrance of the channel, measurements of the pressure as a function of the phase at the rear of the channel and at a position nearer to the entrance should show a shift in the phase at which the maximum pressure occurred. This was verified in the case of the long rectangular channel at an interruption frequency of 250 cycles per second. At position 5 the maximum pressure within the cycle was obtained at the 180° phase, and at position 3, 6 inches nearer the entrance, the maximum pressure occurred at the 225° phase. The time required for the propagation of sound from position 5 to 3 was approximately 0.00045 sec. whereas at the frequency of interruption employed, the time interval between the 180° and the 225° phase measurements was 0.0005 sec.

When the channel exit is suddenly thrown open, the pressure behind the disc falls, the magnitude depending upon the speed and amount of opening of the disc. If the chamber could be thrown open instantaneously, the pressure would fall to that obtaining on the discharge side. This is undoubtedly the cause of the low pressures obtained within the long rectangular channel at 0° phase at 250 interruptions per second.

At the lower frequencies of interruption the graphs show a very sharp fall in pressure from 180° to 225°, reaching a minimum value usually at a 270° phase. At the higher frequencies of interruption subsidiary maxima points occur, in the majority of cases, at a 315° phase.

It is to be emphasized that the above discussion based on Gibson's treatment of the water hammer effect in an hydraulic pipe line is concerned only with the sudden stoppage and the sudden initiation of motion from an initial static condition; i.e., the continual repetition of the closing and opening of the channel is not considered.

Numerous suggestions for further work in this field are contained in the thesis; among them being suggestions for other types of valves and for methods of measuring instantaneous velocities. Of most importance at the present time, however, is the continuation of this work at high air speeds, which could be obtained, possibly, by placing the apparatus in an airplane or in a high speed wind tunnel.

The results of the complete investigation indicate clearly that high compressions should be possible at high air speeds, particularly with the proper number of interruptions of the air stream per second. In general it can be said that, in the case of all channels, pressure variations took place when the exit end of the channel was rapidly opened and closed; that these pressure variations were much greater at the exit end of the channel than at the entrance; were greater as the rate of opening and closing of the exit was increased up to the optimum frequency of interruption; and finally that an increase in air speed caused a much greater rate of increase in the air pressure at the rear of the chamber than at the entrance. (The results shown in Table I do not appear to be in complete agreement with all of the above conclusions since all valve positions were located, in this channel, in the rear half of the channel.)

A striking result of this work is that not only is high pressure obtainable in a straight tube into which air is steadily blown, as a result of rapid interruptions, but a partial vacuum is also producible by direct blowing, without involving Bernoulli's theorem, as is the case with most vacuum effects.

Institute Notes

The members of the Institute in Washington held their first meeting on April 5 at the Cosmos Club. Mr. D. R. Pye, Deputy Director of Scientific Research of the British Air Ministry and author of "The Internal Combustion Engine" was guest of honor at a dinner after which about one hundred members heard him deliver a lecture on "Aircraft Engines, a Forward Look." A digest of the lecture and the discussion will appear in the next issue of the Journal. Dr. George W. Lewis, Vice President of the Institute, presided. Among those present were: E. L. Bass, Technical Director, Shell Oil Companies; Henry Berliner; S. S. Bradley; Dr. Lyman J. Briggs; Paul Brockett; Dr. W. G. Brombacher; Lester D. Gardner; H. K. Cummings; Dr. Hugh L. Dryden; Commander Garland Fulton; J. H. Geisse; W. R. Gregg; Major Roy M. Jones; Carleton Kemper; Captain Frank R. McCrary, U.S.N.; Lt. Comdr. T. C. Lonnquest; Otto Praeger; Group Captain G. R. M. Reid; H. J. E. Reid; Lt. Clyde W. Smith; Starr Truscott; A. M. Rothrock; Dr. L. B. Tuckerman; Dr. Max Munk; John F. Victory; Brig. Gen. O. Westover; Commander A. D. P. Mark Wardlaw; E. P. Warner.

Squadron Commander James Bird, Director of Supermarine Aviation Works and Vickers (Aviation), Ltd., was guest on April 24 at the usual Wednesday luncheon of members of the Institute held at the Aerodrome. Twenty-five members attended and many were old friends of the guest. Members visiting New York find these informal luncheons a pleasant place to exchange views on current progress in aeronautics.

On Friday, April 26th, a meeting was held by the Pacific Coast Section of the Institute at the Athenaeum of the California Institute of Technology, Pasadena, California. After an informal dinner the following papers were presented:

"Aircraft Spot Welding," Bruce Burns (Northrop Corporation).

"Airplane Hydraulic Systems," Harold Adams (Douglas Aircraft Co., Inc.)

"Aircraft Stress Analysis Problems," Carl Stryker (Curtiss-Wright Technical Institute, Glendale).

E. L. Bass, Director of Aeronautical Research of the international group of Shell petroleum companies, was guest speaker at a meeting of the Institute held at the Physics Building, Columbia University, on April 15th. Before the meeting, about fifty members attended a dinner for Mr. Bass at the Columbia Faculty Club. The subject of the lecture was "A Discussion of a Possible Solution of the Lubrication Oil Problem of Aircraft Engines of High Output." Abstracts from the lecture and discussion will be printed in the next issue of the Journal.

The Institute has been accepted as an Affiliated Society in the American Society for the Advancement of Science. The Secretary of the Institute, Lester D. Gardner, has been nominated as a representative on the Council of the Association.

Four members of the Institute were appointed by Dr. Karl T. Compton, chairman of the Science Advisory Board, National Academy of Sciences, to serve on a committee of eight to make a survey for the government of airship construction and to make recommendations as to the future of the airship. They were: William F. Durand, professor emeritus of mechanical engineering, Stanford University; Theodor von Kármán, professor of aeronautics and Director of the Daniel Guggenheim laboratory, California Institute of Technology; Alfred V. de Forest, associate professor of mechanical engineering, Massachusetts Institute of Technology; William Hovgaard, professor of naval design, Massachusetts Institute of Technology.

Dr. J. C. Hunsaker, Honorary Fellow of the Institute and Editor of the Journal of the Aeronautical Sciences, was elected a member of the National Academy of Sciences.

The membership of the Institute on May 6, 1935, had reached 1003 and is gaining every month. Members are reminded that the growth of the Institute has been the result of their suggesting membership to persons whom they believed to be eligible. No one is ever invited to join until after the Admissions Committee has passed upon his qualifications. The Secretary of the Institute will furnish application forms and pamphlets to anyone at the request of a member.

Professor Richard V. Southwell, of Oxford University, and a Scientific Member of the Institute, will deliver a course of lectures at the University of Michigan during a summer course. Professor Southwell is Professor of Engineering Science and has made contributions to the structural analysis of space frameworks, particularly the lighter-than-air ship. He was in charge of experimental work at the Royal Aircraft Establishment during 1918 and later with the N. P. L. He is a Fellow of the Royal Aeronautical Society. Professor Southwell will lecture on Applied Elasticity. Professor S. Timoshenko will give a course on the History of Mechanics; Dr. F. L. Everett on Fatigue of Metals; Professor E. L. Eriksen on Theory of Slender Bars and Thin Plates; Professor M. J. Thompson on Advanced Fluid Mechanics; Professor L. C. Maugh on Rigid Frame Structures, and Professor S. Timoshenko on Vibration Problems in Engineering. The courses will be given between July 24 and August 16. A Seminar will be held at which guest Professors including Dr. Th. von Kármán will discuss various problems of Engineering Mechanics.

The New Grade of Student Membership

The Council of the Institute recently requested from several of the members of faculties of universities and schools, where aeronautical courses are given, their suggestions regarding the creation of a grade of student membership in the Institute. As the replies indicated a desire for such a grade, the Council, at its last meeting, added a new membership classification for students. Other organizations receive $3 a year from students, send them a publication and, we are informed, usually give back practically all the dues in the form of prizes, expenses, grants, etc.

The Council of the Institute decided that it would not follow this plan, but would charge the student only a part of the net cost to the Institute of handling his membership. The following addition was made to the By-Laws:

"Student Members shall be undergraduate or graduate students attending those universities or schools which have on their faculties at least one member of the Institute of the Aeronautical Sciences who recommends them for membership. Any Student Member may continue in that grade for one year after he is no longer in attendance at a university or school. During that period he may apply for membership in another grade, and, if elected, he need not pay any initiation fee."

The dues for student members have been placed at a nominal rate of two dollars a year.

While the Institute does not wish to duplicate existing student aeronautical societies or student branches of the engineering societies, the Council will be glad to receive any suggestions as to having any existing student organizations affiliate as a group. It is believed that the situation in each university and school is different. The Council wishes to cooperate in the most helpful manner possible.

Necrology

Richard F. Hoyt, an Industrial Member of the Institute, was born at Revere, Massachusetts, July 3, 1888, and died at the Doctor's Hospital, New York, on March 7, 1935. He was a graduate of Harvard in 1919 (*magna cum laude*) and immediately entered the investment banking field with Hayden, Stone & Co., of which firm he became a partner nine years later. His enthusiasm for the industrial development of aviation led him to play a leading part in post-war aviation financing and had a profound influence on the subsequent form of many American aeronautical enterprises.

During the war he was assistant to Colonel J. G. Vincent at McCook Field. Soon afterwards he became identified with the original Wright-Martin Aircraft Corporation in 1918, later the Wright Aeronautical Corporation, which was eventually merged with the Curtiss Aeroplane and Motor Corporation into the Curtiss-Wright Corporation, of which Hoyt became Board Chairman.

At the time of his death he was an officer or director of a score of corporations in which Hayden, Stone & Co. was concerned but his chief interest was always directed toward aviation. The Curtiss-Wright union was probably the most significant event in the development of the new aviation industry because it stimulated similar groupings of industrial units making many types of airplanes and engines. In the wave of merging, expanding and regrouping of units, prior to 1929, the Curtiss-Wright group in which Hoyt was a prominent figure, absorbed many other manufacturing concerns and was a participant in the organization of several important air transport companies.

Hoyt's part in the organization of the industry cannot yet be fairly appraised but it may be noted that companies which he organized were not burdened by interest bearing obligations and are surviving the collapse of the aviation boom largely by virtue of their strong cash position due to common stock financing.

In his hours of recreation from a busy life, Hoyt was devoted to private flying, motor boating and yachting.

Personnel Opportunities

The Personnel Bureau serves organizations seeking to employ aeronautical specialists as well as individual members. The Bureau has been the means of arranging several very successful connections for members.

Any member or organization may have their requirements listed without charge.

Wanted

Graduate Engineer and a high type business executive who can fit into foreign and domestic aircraft sales together with administration of business, and other assignments from the President of the Company. Write Box 22, Institute of the Aeronautical Sciences.

Aeronautical Engineer who can write technical articles with some degree of human interest. Write Box 23, Institute of the Aeronautical Sciences.

Graduate Aeronautical Engineers with some practical experience, capable of doing design drafting and carrying through designs. Write Box 27, Institute of the Aeronautical Sciences.

Junior Engineer with engine experience in established engine concern—for accessory development work. Write Box 30, Institute of the Aeronautical Sciences.

Available

Aircraft Engineer and designer. Eighteen years experience. Specialist in application of Aerodynamics to design. Designer of some of the most successful American high speed racing and military aircraft. Write Box 26, Institute of the Aeronautical Sciences.

Connection wanted with aviation company as foreign representative. Applicant graduated 1927; four languages. Write Box 28, Institute of the Aeronautical Sciences.

Graduate Engineer wishes to make new connection. In aviation since 1917. Last seven years in full charge of design projects, including stress analysis, with leading aircraft companies. Write Box 29, Institute of the Aeronautical Sciences.

Letters to the Editor

In "Aircraft Engine Performance with 100 Octane Fuel" by one of us (F. D. Klein) and published in your March 1935 issue it is stated that the response of pure iso-octane to additions of tetraethyl lead is low. It is further stated that an addition of 3 cc of tetraethyl lead per U. S. gal. to iso-octane only results in a gain equivalent to three octane members when tested in accordance with Air Corps Specification 3566.

In the discussion following the paper Dr. Edgar pointed out that the three octane number gain obtained from 3 cc of lead might actually represent a very large gain in fuel value since it was known that the size of an octane number rapidly increased at the upper end of the scale. Dr. Edgar also pointed out that even if successive octane numbers were of sensibly uniform size at the 100 end of the scale, an increase of three might represent a substantial gain in view of the increases in performance reported for 8 octane numbers between 92 and 100. It was also stated in the discussion that a test program was in progress to determine the lead response of commercial iso-octane in a full scale supercharged engine. This work has now been carried out as a result of cooperation between the Lycoming Mfg. Co., the Material Division, U. S. Army Air Corps, and the Ethyl Gasoline Corp.

The engine tests were carried out by the Lycoming Mfg. Co. using a supercharged aircraft engine cylinder of normal type having high wall temperatures. The intake air at the point where it entered the induction system had a temperature of 203° F. and this in conjunction with the cylinder wall temperatures attained subjected the fuels tested to extremely severe conditions. The method of test involved raising the air pressure at the entrance to the induction system until the maximum possible power output for each fuel tested was reached. At each boost pressure on each fuel tested, specific fuel consumption and spark advance were adjusted to the conditions giving maximum power. Three fuels were tested as follows: (a) leaded aviation gasoline meeting the requirements of Air Corps Spec. Y 3557-G, (b) commercial iso-octane and (c) commercial iso-octane plus 3 cc of tetraethyl lead per U. S. gallon.

The aviation gasoline used had an octane number of 92 by Air Corps Spec. 3566 and 87 by the C.F.R. Motor Method.

The commercial octane had the following physical and chemical characteristics:

Specific Gravity	.692
Initial Boiling Point	210.4° F.
90% recovered	210.7
96% "	211.1
End point	217.4
Unsaturation	3% or less

The clear commercial octane had an octane number of 99½ by A.C. Spec. 3566 and 99⅓ by the C.F.R. Motor Method.

The maximum power outputs, reached for the three fuels, and the corresponding manifold pressure expressed as percentages of the maximum obtained with the aviation gasoline were as follows:

	Power	*Absolute manifold pressure*
Y 3557 G	100%	100%
iso-octane	117%	114%
" plus 3 cc $PbET_4$	134%	128%

In the case of both the aviation gasoline and the clear iso-octane violent audible knock was occurring at maximum power and further increase of boost pressure caused a power decrease rather than an increase. In the test of iso-octane plus lead the maximum possible power output as limited by the fuel was not reached since, at the capacity limit of the supercharger, no evidence of audible knock or any other distress could be observed.

The behavior of the three fuels tested is shown graphically in Fig. 1. In all the points plotted in Fig. 1 the spark

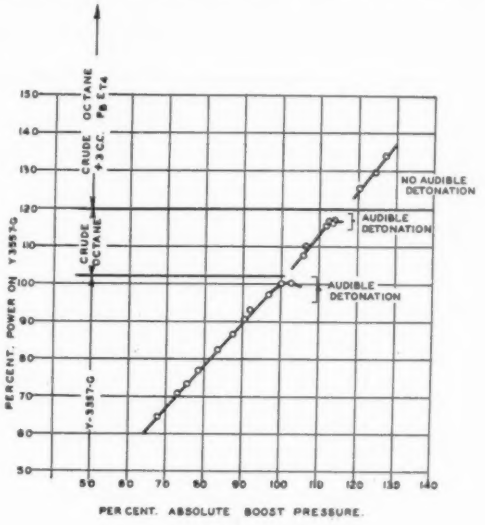


FIG. 1.

advance was 35° except in the case of clear iso-octane where it was retarded to 30° at the highest manifold pressure on account of excessive knock. In the case of iso-octane plus lead at the highest manifold pressure obtainable, spark settings of both 35° and 40° were tested. Since the available manifold pressure would not cause signs of distress, increase of spark advance was tried as a means of producing fuel breakdown but resulted in no change of engine performance.

As evidence that the lead response of octane was not determined under abnormally favorable conditions it may be mentioned that the indicated specific fuel consumption was .425 lb. per hp.-hr. at the maximum output developed on iso-octane plus 3 cc of lead.

It is clear from these data that lead in iso-octane is highly effective in increasing engine performance whatever the lead response in terms of octane numbers as measured by conventional knock-testing methods may be.

The data also indicate that iso-octane plus tetraethyl lead can be used to advantage with very high mixture temperatures. It is stated in the above mentioned paper that intercoolers are desirable for use with 100 octane fuel and highly supercharged engines. While there is no question of the correctness of this conclusion, nevertheless it does appear that, although supercharger intercoolers are desirable, they are not essential to obtain considerable advantage from fuels of 100 octane number or over.

VAL CRONSTEDT
R. N. DUBOIS
F. D. KLEIN
S. D. HERON

April 24, 1935.

I was very interested in the article by Mr. Otto Koppen in the Journal for January, 1935, dealing with lateral control at high angles of attack. As the editor remarks in the footnote to the article, the conclusion at which the author arrives appears to be quite contrary to practical experience. The subject of control at or beyond the stall is of great complexity and involves the consideration of matters like asymmetric stalling due to two different regimes of flow existing simultaneously on left and right wings at the same incidence. But I believe it should be possible to arrive at broad general conclusions on the lines of Mr. Koppen's study. I think, however, that it is essential to include in the control movement a part which is a function of the *rate of bank* as well as the part which is a function of the angle of bank at any instant. The question of lag in the application of the control is also important.

Mr. Koppen has taken derivatives for 16° incidence, before the aeroplane is really properly stalled, or at any rate before autorotation has set in. If autorotation is set up then a control which in effect introduces a damping of the rolling motion is absolutely essential to maintenance of a steady flight path. As for the sign of the yawing moment accompanying the rolling moment given by the control, there appears to be a reason in general for making it of the same sign as the rolling moment; and this is probably because the yawing moment introduced by the motion of the machine itself when it starts to roll is generally such as to increase the accompanying rate of yaw, and so increase the rate of roll, leading to the spiral and spin if unchecked. But the yawing moment given by the control must have an element which depends on the *rate of roll* if it is to be effective.

I am making a theoretical study of this question at the present time and hope to report on it in the near future.

L. W. BRYANT

National Physical Laboratory,
Teddington, England,
April 9, 1935.

The Institute's Aeronautical Index

Since the President of the Institute reported at the Annual Meeting that an aeronautical indexing project was being prepared at the Skyport, substantial progress has been made. About fifteen thousand cards have been filed, and within a month it is hoped that thirty thousand more will be added so that the index will take its place as one of the largest and most complete indexes of aeronautical literature in the world.

To Sherman M. Fairchild who has provided filing cabinets capable of holding eighty thousand cards, the Institute members are indebted. To M. I. T., Cal. Tech., N. Y. U., Georgia Tech., the N. A. C. A., the Aeronautical Chamber of Commerce, The Manufacturers Aircraft Association, the Library of Congress, U. of California, the Glenn L. Martin Co., The Engineering Index, the United Aircraft Corp., The Curtiss Aeroplane and Motor Company and General Aviation, who have cooperated with suggestions and indexing information, the Institute expresses its appreciation.

Reginald M. Cleveland, the Aviation Editor of the New York Times, printed on Sunday, April 28th, a lengthy article on the index, part of which follows:

"An undertaking which it is hoped will prove of immense value to aviation is going forward, it was disclosed last week, at the Skyport, the home of the Institute of the Aeronautical Sciences at Rockefeller Center. This is the preparation of a complete index of aeronautics, covering every phase of this new science. From files of technical magazines, from books and from newspapers unemployed aeronautical specialists, filing clerks and typists furnished by the Works Division of the Emergency Relief Bureau of New York, already have written thousands of cards. A total of at least 100,000 is expected to be reached.

"This index, which covers pictures as well as descriptions of planes, parts, airports and all phases of aeronautics in addition to biographies of the men concerned with the industry, will be available to anyone interested in securing information on aeronautical subjects. Cooperation has been furnished to the Institute by the Massachusetts Institute of Technology, New York University, the California Institute of Technology, and many other institutions.

"Until this work was begun it was practically impossible to locate aeronautical information without a long search of many books, magazines, and newspapers. When this index is complete, it will be possible to find a picture of almost anything that has happened in aviation or of any aeronautical product. We can give anyone a complete list of articles written on practically any aeronautical subject. All aeronautical books will be indexed also.

"Dr. Merle S. Ward, who supervises the project for the Works Division, said, in speaking of the index, that he

thought it was unique in that it took a highly technical branch of science and, by a simple breakdown of the various activities was making it possible to assist aeronautical experts, writers and investigators to find what they wanted without hours of wasted searching.

"The main divisions of the index relate to persons, aircraft and engines. If a picture or a biography of anyone who has been prominent in aviation is wanted, it can be located in a simple alphabetical file. Practically every airplane or aircraft engine will be found indexed by country, manufacturer and type. Such subjects as accidents, aerology, aerobatics, airports, airships, armaments, avigation, balloons, gliders, maps, as well as hundreds of others, will all have their special files.

"Separate headings under the subdivision airplane parts, for example, include fuselage, landing gear, propellers, cockpit, wings, ailerons, slots, wheels, floats, pontoons, dope, chairs, interior furnishings, and cowling.

"Major Lester D. Gardner, secretary of the Institute, who is acting as technical director of the project, said that he knew of no existing index comparable in extent to the Institute's file. Duplication of past classifications is avoided by including them as a part of the broader plan now in progress. He is emphatic in his opinion that this kind of work for the unemployed is not only providing them a living wage and giving them interesting work to do, but will have a permanent value long after the present period of unemployment has passed."

Endowment Fund

The Royal Aeronautical Society established an Endowment Fund several years ago. The Council of the R.Ae.S. hopes that it will grow to $250,000 in a few years. Already gifts, amounting to about $50,000, have been received.

The Institute has established an Endowment Fund with similar hopes. No better statement of its appeal can be made than to quote the opening words of an appeal from the R.Ae.S. for the Fund.

"Let it be made clear at once that this appeal is addressed, under an urgent sense of the important matters it concerns, to all those, irrespective of rank or wealth, occupation or position, who are interested directly or indirectly in British Aeronautical Science, or have faith in Aviation as a safe and rapid method of transportation, essential alike to the development of inter-communications within the Empire and the world at large. In its present renewed, and it is hoped more forcible fashion, the appeal is rightly brought at the outset to the notice of all past and present members of the Society—scientists, research workers, designers, engineers, pilots, and the rest—to whose active cooperation and loyal support the prestige of the Society is so largely due. But it is addressed also, and with equal confidence of support, to that wider circle of the general public, to the more far-sighted and public-spirited men of business and affairs, who must realise the imperative need of maintaining, in the national interest, the highest possible standards in Aeronautical Science and research."

At the last meeting of the Council the following section was added to the By-Laws of the Institute.

ENDOWMENT FUND

"The Endowment Fund shall consist of such real and personal property as may be from time to time accumulated or acquired by deed, gift, bequest, legacy, or by transfer from the general funds of the Institute. The Endowment Fund shall be administered by the Council, either directly or through a committee to be known as the Finance Committee. Property in the Endowment Fund may be sold from time to time and the proceeds together with monies received into that fund invested and reinvested in such securities as may be selected by the Council or the Finance Committee having primary regard to the safety of the principal, but nothing herein contained shall restrict the investment of the funds of the Endowment Fund to securities legal for the investment of trust funds under the laws of the State of New York.

"The creation of this Fund is undertaken to secure a substantial endowment, the income from which shall permit the Institute to carry on its work to greater advantage than would be possible when relying solely upon income from membership dues. The Endowment Fund is to provide means for the Institute to participate in joint meetings with other societies to provide prizes, medals, or other special recognition for those members of the Institute who might contribute noteworthy papers to the Journal or technical meetings of the Institute; to assist student members engaged in aeronautical study; and to extend the work of the Institute, thus making membership more attractive."

The following forms for gifts and bequests to the Endowment Fund submitted by the Secretary were approved:

For Gifts

The Institute of the Aeronautical Sciences, Inc., a New York corporation, having established a fund known as the Endowment Fund, I hereby give the said Institute of the Aeronautical Sciences, Inc., the sum of .... Dollars ($.....), which shall be a part of the said Endowment Fund to be administered in accordance with the By-Laws of the said Institute and the income therefrom being used for such purposes as the Council of the Institute may designate.

WITNESS my hand and seal at ..... this ..... day of ..... 19...

(Signed) .....

For Bequests

I give and bequeath to the Institute of the Aeronautical Sciences, Inc., a New York corporation, having its office in the city of New York, the sum of $....., to be and become a part of the Endowment Fund of that Institute and to be administered in accordance with the By-Laws thereof.

PIONEER
Ring Light

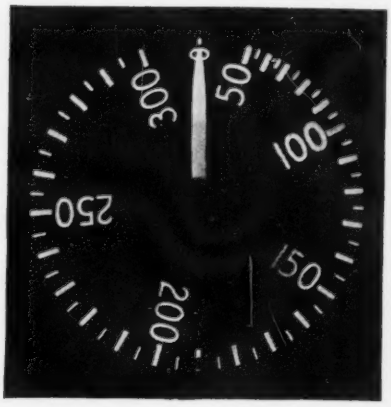
The Pioneer Ring Light incorporates a circular glass rod arranged to lead light around the circumference of any standard instrument dial.

The ring light is so constructed that light is distributed evenly over the dial markings and the end of the pointer. The general effect is that of indirect lighting, there being no glare regardless of the angle from which the instrument is observed.

A small lamp easily replaceable in flight is the source of light. All electrical connections are entirely outside of the instrument case and are unaffected by vibration.

A standard cover glass is used which carries no electrical connections and may be replaced without disturbing the lighting system. The wiring up to the lamp is completely shielded.

The intensity of illumination may be varied to meet requirement by employing a rheostat.



Actual unretouched photo of Pioneer Ring Light.

Ring Lights are available integral with cases of new instruments or in the form of Adapters for application to standard instruments...requiring no major change in the instrument cases.

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